



PERSPECTIVE

Lesson Study: Enhancing Mathematics Teaching and Learning

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The Centre was set up to provide help and support to mathematics teachers around the world through research, development, evaluation and dissemination of good practice in mathematics teaching and learning.

He has directed three international longitudinal comparative projects concerned with making recommendations for enhancing the teaching and learning of primary and secondary mathematics and the teacher training of mathematics teachers.

He is passionate about enhancing mathematics teaching for all pupils and students, whatever their ability and sees mathematics as a lively, relevant and interesting topic to study at all levels.

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After gaining a first class honours degree in Mathematics from University College, London, he progressed rapidly as a teacher and was soon appointed as Head of Department at the age of 25 in a large school in Derbyshire.

He moved to a second career, setting up his own business in the motor industry, including making replica E-Type Jaguar cars but returned to teaching in the early 1990s.

He was appointed a mathematics teacher at **Bishop Luffa School**, a comprehensive school in Chichester in 1993, and was promoted to an Advanced Skills Teacher (AST) in 1996 and Head of Department in 2001.

He is passionate about his outreach work, promoting 'good lessons that work' in the county. He is likewise committed to team work and under his leadership mathematics results at his school have risen to a truly remarkable level (for example, 85% Grade A*–C in GCSE in 2008).

1. Introduction to Lesson Study

“Many have credited the steady improvement of Japanese elementary mathematics and science instruction to their teachers undertaking ‘lesson study’.”

Lesson study, a form of collaborative practice, is a school-based professional development initiative that aims to enhance teaching and learning through the methodology of professional sharing of practice.

A group of teachers collaborate, identifying a research theme or overarching aim that is student centred and relates to the school's vision of what qualities they wish to encourage in their students.

Having decided on the overarching aim, the group meet to plan a research lesson that will bring this aim to life.

The team now follow the process summarised below:

- choose a suitable topic to study;
- identify the goals of the unit of study;
- jointly map out a series of lessons that will achieve these goals;
- identify the key lesson in this series which then becomes the research lesson;
- jointly plan the research lesson;
- one of the group teaches the lesson;
- the others observe the lesson;
- review and reflect on the lesson;
- revise the lesson plan and continue the cycle.

Ultimately the group will produce well-trying and tested lesson plans, contributing to both the chosen research theme and the topic under study.

It is an excellent form of CPD (continuing professional development), which has the potential to make real improvements to teaching and learning that are sustainable and not just short-term gains. This is, in part, due to the fact that the teachers are dealing with their own classrooms and their own students rather than theoretical situations with virtual students, the model used for much CPD work in the past two or three decades.

There is no doubt that lesson study has the potential to radically transform schools into learning environments in which teachers, working collaboratively, can investigate, share and verify what works well for their students. (We will use the term ‘student’ for all learners in this manual, whatever their age.)

It will, however, only provide significant enhancements in learning if some key conditions are met. Central to the process are:

- backing by the senior management of the school so that sufficient time is allowed for the group of teachers to meet in order to plan, observe and review the lessons, on a regular basis – this is essential;
- teachers must be open minded about different strategies and prepared to experiment and innovate and to learn from their colleagues in an ethos of collaboration and cooperation (not competitive and not appraising);
- schools must form links with ‘knowledgeable others’ and teachers must be open to alternative methods of teaching.

These points will be dealt with later in this manual of good practice for lesson study. Firstly though, we will look briefly at the history of lesson study.

Japanese Lesson Study

Many have credited the steady improvement of Japanese elementary mathematics and science instruction to their teachers undertaking ‘lesson study’. This is a process in which teachers jointly plan, observe, analyse and refine actual classroom lessons, called ‘research lessons’. It is a widespread initiative in Japan, and an integral part of their pre-service teacher training programmes. In addition, lesson study forms the basis of continuing professional development for teachers within their own school and sometimes across schools within a district.

Lesson study is based on three underlying principles:

- teachers learn best from and improve their practice by seeing other teachers teach;

- teachers who have developed deep understanding of and skill in subject matter pedagogy should share their knowledge and experience with colleagues;
- teachers should cultivate students' interest and focus on the quality of their learning.

In primary schools, almost all Japanese teachers take part in lesson study and are involved in two or three cycles per year.

In each case, the lesson study involves:

- groups of teachers thinking about the long-term goals of education, and in particular identifying gaps between the 'ideal' and 'actual' qualities of their own students;
- adopting a long-term research theme that focuses on closing one of these identified gaps;
- focusing on the goals of a particular subject area, unit or lesson that has been identified by the group for a specific reason. (For example it contains common misconceptions that have been identified from student data; it is an area that teachers find difficult to teach; it is a new topic or it simply fits best within the lesson study schedule.);
- the group carefully planning the classroom 'research lesson' that brings to life both the topic and the longer term aims for the students; they might adapt published lessons or use previous lesson plans;
- undertaking a detailed study of how the students respond to these lessons, including their learning, engagement and interaction;
- revising the lesson plans and teaching the lesson again, revising again if necessary.

It is common practice in Japan for a group member to take detailed notes of all of the meetings. In this way all of the group's work is available for future reference and in particular the thinking that lies behind their decisions.

At the end of the school year, schools often publish a report on their lesson study activities. These reports contain detailed notes of all of the meetings as well as the lesson plans, photographs and a DVD of the lessons. In this

way, the group's knowledge can be shared with a wider audience.

You can find further details in **references (1) and (2)**. Our own experiences indicate that lesson study is well established in the primary sector but appears not to be quite so well used in the secondary sector. Catherine Lewis, author of more than 40 publications on elementary education and child development, who has made detailed studies of lesson study, states that:

'Japanese teachers say that the most powerful part of lesson study is that you develop the vision to see children. So you're really watching how children are learning, and learning to see things that you didn't see before: their thinking and their reactions.'

It is important to emphasise that unlike teachers in the West, Japanese teachers have a long tradition of opening their classrooms to other teachers as a means of sharing practice. In addition, for almost 50 years Japanese schools have organised their own school-based, in-service training that brings together the entire teaching staff to work on a school wide goal that all teachers have agreed is of critical importance to them.

A further difference between Japan and the UK is that schools and universities work much more closely together. Lesson study groups in Japan are supported by 'knowledgeable others', often from local universities, who provide expertise in the actual lesson study process, help to plan the lesson and provide curriculum expertise.

As mentioned above, Japanese teachers are expected to 'cultivate students' interest' in mathematics. In general this is achieved by planning lessons around problems or tasks that encourage reflection and communication. As Kishimoto and Tsubota (**see reference (3)**) explain, most mathematics lessons in Japan encourage students to take an active role in constructing their own mathematics by communicating with one another; students are encouraged to develop a belief in their own ability to learn and to think. These lessons often feature the 'Open Approach'. An explanation of the 'Open Approach' is given in **Appendix 8**.

USA initiatives in Lesson Study

The adoption of the Japanese lesson study model was promoted in the USA by Stigler and Hiebert (**reference (4)**) using the TIMSS (Third International Mathematics and Science Study) results and video evidence.

The first lesson study group was formed at the UCLA Lab School in 1993 and this was the start of great interest in this technique for improving practice in schools in the USA. It was not just the work of Stigler (**see reference (5)**), which has resulted in a commercial operation for lesson study, funded by the educational publisher, Pearson (**reference (6)**); educational researchers based at Columbia University (**reference (7)**) have provided detailed information and templates for implementation and it is estimated that over 30 states now have lesson study groups, with over 2000 teachers involved in lesson study.

The model recommended usually has the following stages:

1. **choose a research theme:** a group of teachers agree a research theme, often school/college-wide and involving student skills or attitudes that the school or college wishes to promote;
2. **focus the research:** the group chooses a unit of study to focus on and considers the overall research theme in this context;
3. **create the lesson:** the group selects a lesson or lessons within the unit to concentrate on, using a lesson planning template to plan lessons jointly;
4. **teach and observe the lesson:** the lesson is taught by a member of the group and observed by the other members; the focus of the observation is on student thinking and responses rather than the teacher's ability;
5. **discuss the lesson:** the group gets together, usually on the day of the lesson, to discuss the outcomes of the lesson and their observations;
6. **revise the lesson:** revisions are made to the lesson, based on the observations and one of the group is selected to teach the lesson again; the cycle of observation, discussion and revision is repeated if necessary;
7. **document the findings:** at the end of this process, the group produces a report that outlines what they have learnt with regard to the research theme and goal;
8. **present the findings:** the group presents its findings to other groups and/or conferences. The length of the process varies but it is not uncommon to take up to a year or two. The size of the group also varies but typically involves four to six teachers, as well as an administrator and external expert. The whole process is thought of as a bridge formed by teachers working together and collaborating across the curriculum rather than in isolation.

Catherine Lewis, one of the promoters of lesson study in the USA, has produced a useful handbook for teachers (**reference (8)**).

In this book she points out that,

'Lesson Study seems a simple idea. If you want to improve instruction, what could be more obvious than collaborating with fellow teachers to plan, observe and reflect on lessons?'

But she goes on to warn that, *'While it may be a simple idea, lesson study is a complex process.'*

The rapid expansion of lesson study in the US has caused some concern amongst educators such as Chokshi and Fernandez (**reference (9)**) who point out *'since deep knowledge about lesson study is rare in the U.S., it is likely that some of these groups have an incomplete understanding of this Japanese practice.'* This is supported by Lewis (**reference (10)**) who warns that, *'Premature expertise may pose a substantial threat to lesson study.'*

Yoshida (**reference (11)**) contrasts this with the Japanese teachers' belief that major change requires a ten-year settling in period. He stresses that 'we must be patient.' As Fullan (**reference (12)**) warns, new initiatives 'go nowhere' when there is a lack of 'deep theoretical understanding of the first principles of learning'.

Eastern European model

Although teachers in countries such as Hungary, Poland and the Czech Republic would not

“The key point though is to recognise that this form of training is lesson study in another guise, where the group consists of future teachers and an expert teacher.”

recognise that what they do is lesson study, they have a very informed approach to CPD in which teachers will naturally observe and review lessons with colleagues. It is more of an informal process although a more formal lesson study model is used extensively in initial teacher training.

Here groups of up to eight future teachers work in their first practice with their mentor (expert teacher) in a University Practice School, jointly planning, observing and reflecting on the lessons, with each future teacher taking their turn to deliver the lesson.

These future teachers actually teach very few lessons compared to their counterparts in the UK, but they do help plan lessons and, crucially, have the chance to review and reflect on a large number of lessons. Not only is this regarded as an excellent way of training teachers, it is also a mechanism for self selection. Future teachers who are not suited to the profession will recognise this at an early stage of the training and withdraw voluntarily rather than continue and be failed at the end of the course.

The key point though is to recognise that this form of training is lesson study in another guise, where the group consists of future teachers and an expert teacher. This is very similar to groups of teachers working on lesson study with an outside expert, who will share in the review sessions.

UK initiatives

Whilst collaborative practice has been on the school agenda for some time, it has taken longer to become established. We will report on our initiatives with Pathfinder schools for NCETM (National Centre for Excellence in the Teaching of Mathematics) during 2006/07, and further progress through our PPD (Personal Professional Development) work, funded by the TDA (Training and Development Agency for Schools), over the past two years.

Our work on collaborative practice led us towards Japanese lesson study as a proven method of improving the quality of teaching and learning. What rapidly became apparent is that lesson study is indeed a complex process

and can not be seen as a ‘quick fix’. It is hard work. It requires commitment from the head teacher and the teachers involved as well as a willingness to seek outside help. It works best as a whole school initiative where everyone involved is united by a common goal. Our research, however, indicates that the results are well worth the effort.

It is clear, however, that we have not yet reached a ‘tipping point’ where either collaborative practice or lesson study is a regular and sustained initiative, at either primary or secondary.

It should be noted that the National College for School Leadership has promoted this idea and has produced a series of booklets under the umbrella of ‘Networked Learning Communities’ (see **reference (13)** for details).

It should also be noted that the recent EPPI (Evidence for Policy and Practice Information) review (**reference (14)**) on collaborative CPD provides a useful backdrop and encouragement for lesson study.

The main results of the review are summarised in **Appendix 1** although it should be noted that this research was on ‘collaborative CPD’, which clearly included lesson study but has a slightly wider context. Nevertheless, it is instructive to have evidence that this type of collaborative working does in fact produce gains for both teachers and students.

All our comments have so far been generic and not subject based. From now on, we will concentrate on lesson study for enhancing mathematics teaching and learning, but it is undoubtedly true that much of what we report and our guidelines for implementing lesson study will be appropriate for any school subject.

2. Issues in Mathematics Teaching and Learning

“ We believe that the long-term improvement in teaching will depend more on the development of effective models for teaching than on identification and recruitment of talented individuals into the profession. ”

Mathematics education in the UK has undergone many changes in the past few decades in terms of the curriculum, assessment and teaching strategies. Unfortunately, despite all the changes and initiatives, mathematics still seems to be a problem in this country.

Our own research (see **references (15)** for secondary and **(16)** for primary) illustrates that our attainment is well below that of many of our international competitors; it also indicates that there are strategies used by mathematically high-performing countries that could be implemented here.

In the primary phase, these high-performing countries are all characterised by having a strong mathematical foundation put into place so that, for example, topics in algebra are gradually and naturally introduced from an early age.

In the secondary phase, mathematically high-performing countries use homework as an integral part of the learning, with a few questions set after every lesson to both reinforce the lesson and bridge the gap to the next lesson. The work is reinforced at the start of the next lesson with students providing solutions on the board for class discussion.

In both the primary and secondary phases, there is a highly interactive style of teaching with, at its highest level (and on a regular basis), students demonstrating and articulating their solutions to problems on the board in front of the class. In effect, the students become the teacher for a minute or two and, as a consequence, they are more likely to remember what they have learnt and other members of the class will gain through hearing an explanation from one of their peers.

Earlier we mentioned the research of Stigler and Hiebert (**reference (4)**). Not only did they recognise the potential of lesson study but they also highlighted that teaching is actually a cultural activity. They stress that these ‘cultural scripts’ are learned, not by intentional study, but ‘through informal participation over long periods of time’. In other words, most

prospective teachers are heavily influenced by their own experience at school and consequently teach in a similar way to the way they themselves were taught.

They go on to suggest that the problem can’t be solved by simply recruiting better teachers as the problem is much deeper:

‘We believe that the long-term improvement in teaching will depend more on the development of effective models for teaching than on identification and recruitment of talented individuals into the profession.’

The thrust of their argument is that, ‘...teachers follow scripts as members of their culture, and their effectiveness depends on the scripts they use’.

Although they acknowledge that the highest performing countries do not appear to follow the same script, they did notice that all of these countries gave their students some opportunities to solve challenging problems that require them to construct mathematical relationships in order to develop conceptual understanding. In other words, students solve problems not to apply what they have already been taught but to learn new mathematics.

For example, unlike the West where students spend most of the lesson doing a range of examples, many Japanese mathematics lessons are based around one specific problem. Fernandez and Yoshida (**reference (17)**) stress that the choice of problem must encourage flexible approaches and this should be made clear by the way the question is framed. They add that in Japan teachers often emphasise this by asking their students to ‘think about as many different solutions as possible’. A more detailed explanation of the Japanese approach to problem solving is given in **Appendix 8**.

There are also issues that would be difficult to change: for example, we have a curriculum that is constantly and regularly assessed through national testing (although recent government changes have dropped Key

“We are not saying that the teacher, as a professional, should be guided at all times by these suggestions since individual teachers will determine what works best for them and their classes.”

“Because human beings created mathematics, children can create mathematics.”

Stage 3 tests) and much of our teaching is dominated by preparation of our students for the next set of tests. This contrasts to other countries such as Hungary, Finland and Japan that have no national testing until age 18.

Our international research indicates a number of strategies, used by many of the mathematically high-performing countries, which could provide the focus for practitioner research in the classroom. These strategies, given in **Section 2**, would contribute to an improvement in mathematics teaching and learning so that students would:

- be on task, making continuous progress in mathematics
- enjoy their mathematics lessons
- become mathematical thinkers.

We are not saying that the teacher, as a professional, should be guided at all times by these suggestions since individual teachers will determine what works best for them and their classes.

We do though recommend that teachers should at least consider ways in which the teaching and learning of mathematics can be enhanced and we hope these strategies will provide ideas for the focus of lesson study in which professional expertise can be shared.

One of our Pathfinder schools had various inspirational quotes in their staffroom. We particularly liked the quote attributed to Yoshishige Sugiyama,

‘Because human beings created mathematics, children can create mathematics.’

We firmly believe that students’ perceptions of mathematics are formed by the kinds of tasks they are asked to perform on a day-to-day basis.

If students are routinely asked to practise previously described procedures by completing a set of exercises, then they will conclude that mathematics is all about learning and following rules.

If, however, we want our students to see mathematics as something they can create for themselves, we need to give them problems for which they have no known methods. In this way students can be encouraged to learn together through reflection and communication.

We agree with David Tall’s view (**reference (18)**):

‘What is absolutely clear to me is that Lesson Study has genuine benefits that would be of value to us in the United Kingdom, as long as we think reflectively about what it is we are trying to do in teaching mathematics.’

Appendix 2 gives a further, more comprehensive list of possible areas to focus on in lesson study.

3. Feedback from Pathfinder Schools

“The collaborative practice model being advocated by CIMT was based on the informal approach seen in many Eastern European countries but was also similar to the more formal lesson study practised in Japan and the USA.”

In September 2005, with funding from the Esmée Fairbairn Foundation, a group of Pathfinder organisations was formed on a purely voluntary basis from schools which had already been closely involved with CIMT on the Mathematics Enhancement Programme (MEP) or interested schools which had contacted CIMT asking to be involved.

There was a mix of primary, middle and secondary schools and colleges and their task was to undertake a proof of concept study on using collaborative practice through lesson study as an effective form of CPD.

The collaborative practice model being advocated by CIMT was based on the informal approach seen in many Eastern European countries but was also similar to the more formal lesson study practised in Japan and

the USA. The suggested methodology for collaborative practice is summarised in Figure 1 below.

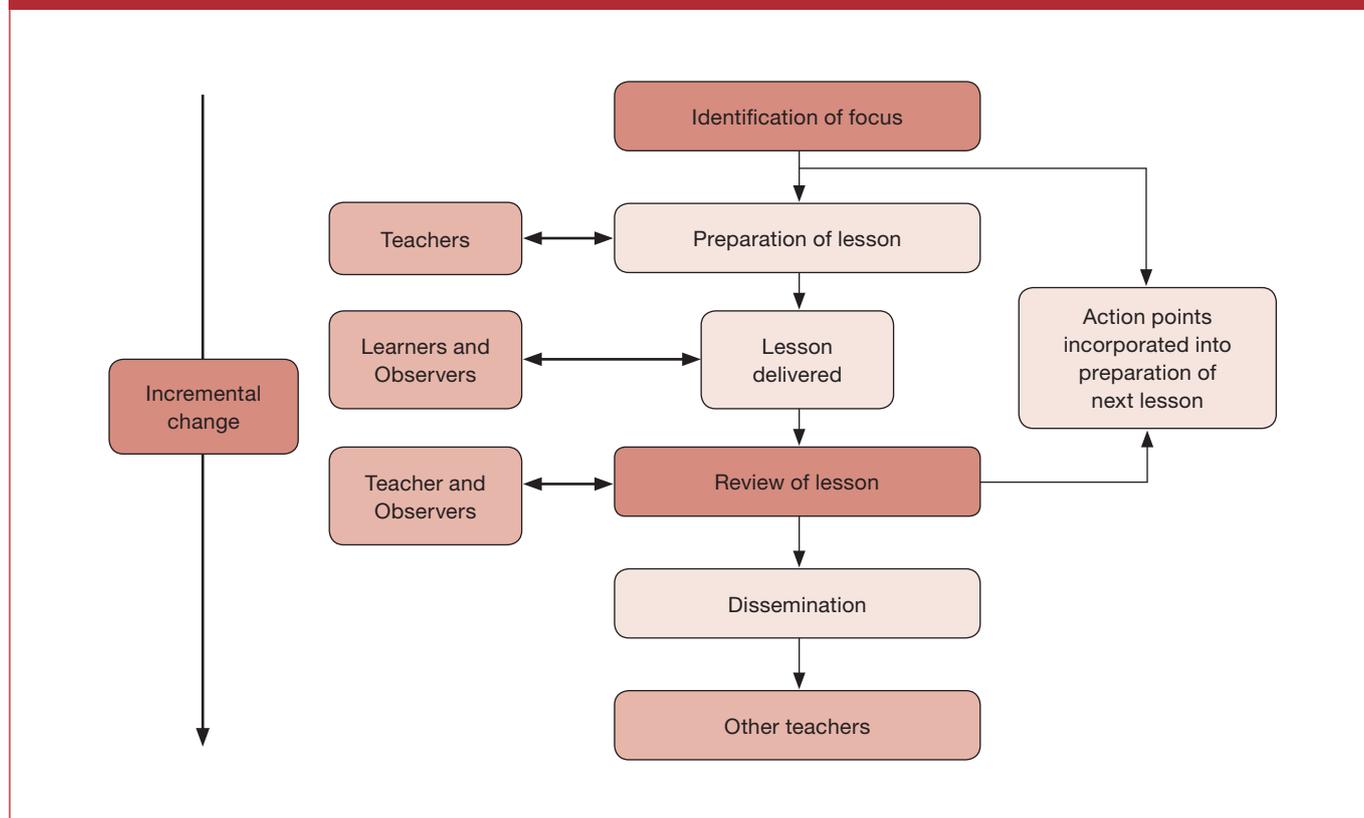
This project has been evaluated through the use of questionnaires for teachers and students, videos of planning sessions, lessons and review sessions and three discussion forums in which teachers from each institution were represented.

Preparation for Collaborative CPD

Before schools and colleges started to implement collaborative practice CPD, at the first discussion forum some protocols were established and agreed on.

- Each school/college had to decide what it wanted its teachers and students to gain from this CPD initiative.

Figure 1: The suggested methodology for collaborative practice



“The major factors hindering implementation were finding time for observing and reviewing and obtaining support from the rest of the mathematics department and from senior management.”

- Collaborative CPD (Lesson Study) should not be confused with monitoring and appraisal.
- Teachers should decide on a focus for each lesson.
- Everyone observing should take notes during the lesson (to aid them during the review session).
- During the feedback:
 - all present should give reviews
 - one person should lead the ensuing discussion
 - the lesson summary form should be completed
 - action points for the future should be agreed, and copies given to every teacher.
- The planning, observation and feedback sessions should be built into the school/college timetable.
- Criteria for observations should come from the teachers themselves and not be imposed, although suggestions and prompts could be made (as on the lesson review sheet provided, **Appendix 6**).

To prevent the dissemination of poor practice, continual evaluation was necessary.

Information had to be collected from teachers and learners about whether collaborative CPD in this form was making a difference to learning.

There was much diversity in the problems that schools and colleges faced initially, how these problems were overcome, the methods of implementation, the personnel involved and the timings of the review sessions. The following section summarises feedback from the questionnaires and discussion forums.

Initial problems

The major factors hindering implementation were finding time for observing and reviewing and obtaining support from the rest of the mathematics department and from senior management. Where there was opposition from senior management or reluctance from staff, CIMT coordinators visited the organisation to explain exactly what

collaborative practice is, its grounding in international research and to persuade them at least to try it out. This was successful in all cases.

In one secondary school, two reluctant part-time members of staff were paid for attending planning and feedback sessions although eventually they wanted to be involved, once they realised how valuable this form of CPD could be.

Planning

Several schools began the cycle of observation and reflection with a collaborative planning session but others could not find the time so staff taught their normal lessons or used, for example, the MEP Lesson Plans (freely available on the CIMT website). The positive impacts of shared planning were that it gave support and confidence to inexperienced teachers (especially NQTs) and generated trust and the feeling of being part of a team from the start. Teachers were able to pool their ideas on how to teach a certain topic and took collective responsibility for the lesson that followed.

One secondary school regarded the planning as being such an important part of the model that they intend to use future departmental meetings for planning lessons, rather than discussing administrative tasks.

A drawback which emerged from the general discussion was that collaborative planning might inhibit individual creativity: a strategy or activity which might have had a positive effect when undertaken by one teacher might not have the same expected effect when delivered by someone else. This led one department to make sure that in future their lesson planning would be more specific and detailed.

In another secondary school, *after* the group of four mathematics teachers had contributed their ideas and planned the lessons, the person who would teach the lesson was chosen (by tossing a coin, etc). This encouraged greater creativity in the planning as well as a shared responsibility for the lesson.

“A video takes a long time to watch and also to edit if the teachers involved are not expert in using technology and does not add a great deal to the review session if it is held straight after the lesson.”

Making Time

Although finding time for lesson study was the main issue for all the Pathfinder schools, CIMT staff who were supporting the institutions were surprised by the creative solutions found to this issue.

A variety of strategies were used to make time for planning, observation and review. Here are some of the solutions implemented:

- using the Year 11 mock exams to free teachers so that they could observe Year 7 and Year 8 classes;
- using internal and external cover (in the latter case, money was used from the CPD budget which would normally fund externally run CPD courses);
- using AST's coaching time;
- using teaching assistants to supervise individual learning;
- using deputy heads and heads to take classes;
- combining classes in the school hall;
- using disaggregated in-service training days for collaborative CPD across the whole school.

Initially it was hoped that PPA (planning, preparation and assessment) time would facilitate the initiative but in practice this wasn't the case, although one primary school put all its teachers' PPA time together on a Friday afternoon and organised enrichment and development sessions for students (e.g. football coaching, music, drama, gardening, etc.).

Using Video

Video was used as a tool not only to allow teachers who could not be physically present to see a lesson, but also to enhance the review. In one school, teachers were given a DVD of the lesson to take home that same day and study in detail, and they were then able to show clips to demonstrate points of learning during the later discussion.

Video also helped inexperienced teachers, who were often so traumatised by the experience of being observed that they

could not remember what exactly happened in their lesson, to gain useful insights. Even experienced teachers learned from watching themselves in action.

Teachers who had used video were able to give some advice on its use, e.g. having a microphone at the front of the class and occasionally zooming in on important board work and students' individual work, as well as on students who were making contributions from their desks. The ideal situation would be to have two people videoing (IT technicians could be involved, or able IT students could do the videoing and editing as one of their projects) but this was not thought to be feasible in the majority of schools/colleges.

Some negative aspects also emerged. A video takes a long time to watch and also to edit if the teachers involved are not expert in using technology and does not add a great deal to the review session if it is held straight after the lesson. Another point of view was that if timings were noted on the lesson review notes, there was no need to edit, just run the video through to the required times.

A further disadvantage of video was that in some cases it inhibited both teachers and students; a normally lively teacher or class might be more subdued than usual, or students who liked being the centre of attention played up to it. However, the more often video was used, the more teachers and students became used to its presence and eventually ignored it.

Observations

Some schools/colleges used the lesson observation sheets provided and some adapted them or used their own methods of note-taking. In two schools, monitoring as part of performance management was also undertaken at the same time. However, it was generally agreed that this was not a good way to build up trust among teachers and that lesson study should be an entirely different entity from monitoring.

It was suggested that whether the lesson was planned collaboratively or not, each observer should be given the lesson plan beforehand

“Most teachers were reluctant to criticise their colleagues and even in unsuccessful lessons the majority of observers tried to be positive.”

so that they knew roughly which point in the lesson had been reached at any time.

The effect on students was discussed and it emerged that on the whole they were happy to be observed by other teachers. In particular, those students who struggle with mathematics felt important because other teachers were interested in them! It was agreed that the model should focus as much on how students learn as on how teachers teach.

Lesson Review

The times at which the reviews took place varied from directly after the lesson to a fortnight later, but it was thought that the interval between observation and review should not be more than six days. The Pathfinders used lunch breaks, twilight sessions or time allocated for departmental meetings for the review phase. Ideally, it should be about 10–15 minutes after the lesson to allow for collection of thoughts and perusal of notes. Of course, where lessons had been videoed, time was needed to enable staff who were not present at the lesson to watch the video.

Most teachers were reluctant to criticise their colleagues and even in unsuccessful lessons the majority of observers tried to be positive. At the same time, it was pointed out that unless ineffective strategies were highlighted there would be no progress. Some NQTs felt unable to give feedback to senior teachers and senior teachers did not value feedback from NQTs, but learning to give and accept criticism in a constructive way became easier as time went on.

Most teachers thought that reflections on the lesson was the most valuable part of the model and many interesting and surprising ideas and insights emerged from them, affecting not only an individual teacher but the whole department and in some cases, the whole organisation. For example, the teachers in one mathematics department realised that they were all too controlling and stifling in their lessons and did not allow enough opportunities for open discussion among students, so they decided that they would try to build this aspect into their

lessons in future. Other action points were: to change the material which had been used in the lesson, to involve or give more support to certain students, to make links to other concepts, to find real-world applications or to use suggested IT programs to enhance future lessons.

One disadvantage of paired grouping for lesson study emerged during the review phase – limited feedback and discussion. In one school, an NQT had been paired with a teaching assistant and although both gained something from the experience, the NQT felt that she would have benefited more from a larger group discussion. In many organisations, paired groupings were all that could be managed in the timetable but such places were encouraged to work towards making their groups as large as possible. In the short term, pairs should intermingle, so that a variety of experiences could be shared.

It was suggested that reviews should also be videoed, as many important points arose from them and should be made available to teachers who had not been involved or, in the case of secondary schools and colleges, to other subject departments.

One important aspect agreed on by everyone was that there must be some focus arising from the feedback sessions to take forward to the next lesson. The cycles of planning, observation and review must build on one another to make progress rather than being stand-alone sessions which were interesting at the time but had no lasting effect on teaching and learning.

Outcomes

All the teachers who took part in the lesson study sessions were very positive about the experience, even if the lessons they had given or observed were not entirely successful, and were enthusiastic about continuing the model because teachers were now involved in discussions about effective pedagogy rather than making judgements on one another. One mathematics department which was already effective and where good teaching was already in evidence wanted to spread this good practice within their department (to

“To be effective, the lesson study sessions should be regular and ongoing and feedback should be obtained not only from teachers but also from students.”

make good teachers great teachers) and to other subject departments and thought that the lesson study model was ideal for this. In one school, the most negative teacher at the start eventually became the most supportive and initial resistance to the model on the part of senior management seemed to disappear. Organisations and mathematics departments were able to build up a file of findings to inform new teachers and other subject departments. In several instances, teachers now *wanted* to be observed so that they could try out something new and have it evaluated by the whole ‘team’.

There are other important points to note:

- To be effective, the lesson study sessions should be regular and ongoing and feedback should be obtained not only from teachers but also from students. Teaching and learning should be seen to be improving slowly but steadily.
- The methodology has been successful in involving all teachers and seems an effective way to influence those ‘hard to reach’ teachers who show little or no interest in other forms of CPD. They recognise that participating in this form of CPD is fun and does enhance their professionalism.
- Classroom assistants, HLTAs (higher level teaching assistants), student teachers, SAS (Student Associate Scheme) students, sixth form helpers, etc. should also be involved in the planning, observing and review cycles so that they feel part of the ‘team’ and have the opportunity to give input from a different perspective.
- Primary schools in the project were able to implement the model more effectively than the secondary schools and colleges because in the former case, the whole school was collaborating and the initiative was backed by the headteacher, whereas in the latter only the mathematics department was involved. In addition, the relationships among staff tended to be more positive in primary schools as they were more used to helping each other and there was generally more trust among staff.
- Although there were instances where learners ‘played up to the camera’, the

majority were pleased to be involved in the training of their own teachers and either did not think it affected their learning or thought it enhanced their learning because of more effective teaching strategies and ideas. They certainly were happier to have teachers they knew in their classroom rather than have a stranger take the place of their teacher.

Evaluation Data

In the evaluation questionnaire, teachers were asked to grade a number of factors from their collaborative CPD on a scale of 1 (very negative) to 5 (very positive). The mean values and standard deviations of these factors are summarised below:

Factor	Mean	Standard Deviation
Enjoyment	3.65	0.70
Mathematical value	3.88	0.70
Pedagogical value	4.24	0.66
Motivational value	3.88	0.99
Lasting effect on teaching	3.82	0.81
Professional development rating	4.00	0.87
Effect on students’ learning	3.47	0.86

They were also asked:

‘What do you think are the pros and cons of collaborative practice CPD compared with external courses?’

Here are representative responses to this question:

Positives

- Focus on our own needs rather than irrelevant needs
- Immediate feedback and implementation
- Stronger department, working together
- More specific to school and departmental needs
- Real sharing of ideas

“Whatever the situation, professional learning has always ensued.”

- Seeing new ideas implemented
- All resources ‘in house’
- Working together in a non-threatening environment
- Reaches all teachers
- Minimises disruption to classes – little or no supply cover needed
- Discussing teaching and learning with fellow professionals
- Good to talk and share ideas
- Focus on good practice in department (often better than outside can provide)

Negatives

- Logistics
- Difficulty in organising and finding time
- Giving quality time to review
- Staff anxiety about being (critically) observed by peers
- Time taken away from other things
- Internal embarrassment and fear of criticism

There were far more positive comments than negative ones.

On the Student Questionnaire, one question was:

‘Do you think it is a good idea for teachers to see each other’s lessons?’

Please give a reason for your answer.’

There was a 100% YES to the first part of the question; here are some representative responses:

It helps teachers to learn.

It gives teachers ideas to try out in their class.

Students can get to know other teachers.

If teachers have to cover a lesson, they know the class and how they learn.

They find out the different ways that students learn.

They find out different ways to manage behaviour.

They see how other teachers explain different concepts.

They can see what level we are all at, our weak and strong points, so if they ever teach me they will know how to help me learn.

*They can find out ways to make us learn **and** how to make maths lessons fun.*

Students behaved better so I enjoyed the lesson more.

If our teacher watches other lessons, she can teach us what she sees.

Because if our teacher is away, we can be sure that we always have a good teacher.

It was a better lesson than usual.

Summary

We were delighted with the responses as well as the positive experiences we had witnessed. Some Pathfinder schools have instigated a lesson study day at their school and invited other schools in their network to attend, while others have visited schools outside their network and a few had visited schools in other countries.

Whatever the situation, professional learning has always ensued. Those who had shared such experiences agreed that lesson study is a very positive experience for all the participants and has even more potential for enhancing mathematics teaching and learning with networking of effective teachers and outside experts in the process.

4. Recommended Practice for Lesson Study in Mathematics

“From our own studies and those of many other educators around the world, we know that lesson study has the potential to transform teaching and learning.”

You will have seen from the previous section that lesson study is a powerful form of school-based CPD. In fact, it is probably the most productive way to enhance and revitalise mathematics teaching and learning in your school. Not only does it deal directly with your students and your classrooms but it also values teachers as professionals. This is in marked contrast to much of the CPD that has been imposed on teachers of mathematics in recent years.

From our own studies and those of many other educators around the world, we know that lesson study has the potential to transform teaching and learning. Yet not all schools will achieve their potential gains as there are many pitfalls and many less-than-perfect strategies which can affect the way it is used. For maximum impact, lesson study has to be combined with rethinking what makes effective teaching in mathematics.

This section deals with what we see as good practice in the implementation of lesson study. We fully recognise that ‘one size does not fit all’ but we do think that there are clear pointers for its effective implementation. We provide here some of the tools needed for the planning, observing and reviewing of lessons. We have divided this advice into four logical stages.

Stage 1: Getting started

This is probably the most difficult stage. Whether you are in a secondary school, where you will have a team of full-time mathematics teachers and perhaps some part-time teachers or in a primary school, where most, if not all teachers, teach mathematics, you need to involve all teachers of mathematics in this process. Although you can, of course, have a sub-group to pilot lesson study, our advice would be to:

- get the agreement of all teachers of mathematics to participate in the initiative.

To help achieve this, you need a leader (e.g. AST Mathematics teacher or Head of Mathematics at secondary, Head Teacher or Mathematics Coordinator at primary) to oversee the initiative and to cope with queries and issues that other staff will raise. This is an important leadership role and it needs to be undertaken by someone who is passionate about mathematics and can lead and inspire others.

We would advise you to:

- select a leader to drive the initiative and deal with issues that arise.

The next process in Stage 1 is to decide on the research theme or overarching aim for the school. It should be student centred and should relate to the school’s vision of what kind of students they want to produce in their school. By preference it should be a whole school vision and be followed over several years.

Lewis (**reference (8)**) suggests that lesson study groups start by thinking about their own students and answering two questions:

- Ideally, what qualities would you like these students to have in five years time (or alternatively when they leave your school)?
- What actual qualities do our students have now?

By comparing ‘ideal’ with ‘actual’, the group can identify meaningful gaps that relate to their own school and their own pupils. Lesson study is the road that the group will navigate to close these identified gaps.

Stepanek *et al.* (**reference (19)**) point out that the overarching aim can also be aligned to existing school improvement goals or school mission statements. She adds that, as the long term success of the study will be measured against this research theme rather than against specific mathematical objectives, it is vital that the team agree on their decision and can justify it to others.

“ You might like to involve teaching assistants in the process as they will clearly have much to offer, although they may wish not to actually teach a lesson in the cycle of lessons to be followed. ”

Guskey (reference (20)) emphasises that effective CPD must link professional development with improvements in student learning. In effect, Lesson study utilises a system of ‘backwards design’ in which your overarching aim defines the changes desired in your students. In other words the changes become the goals that the professional development is intended to achieve.

Guskey goes on to emphasise that professional development needs to be both ‘purposeful and intentional.’ He suggests that this requires three steps.

1. Begin with a clear statement of purposes and goals.
2. Ensure that the goals are worthwhile.
3. Determine how the goals can be assessed.

An example of an overarching aim is included in Section 7.

Consequently our advice is to:

- agree an overarching aim of acknowledged importance for the research focus of your lesson study
- turn your overarching aim into specific, measurable objectives that can eventually be used to evaluate the success of your research lessons over the next three years.

In addition to the school’s wide overarching aim, you may have specific mathematical needs: for example, known underachievement of students in mathematics or a new mathematics scheme being implemented. The choices should be debated and refined for the purposes of lesson study.

Whilst, as with teacher training, the focus could be simply on ‘effective teaching and learning’, our experience has shown that it is probably better to have a more precise focus.

Appendix 2 lists some of the areas that might be used as a focus but this list is not comprehensive and it is better for your focus to emerge from discussions rather than be imposed. Whatever is chosen, you can be

guaranteed that all teachers involved will be working at M (Masters) level as they will be reflecting in some depth on mathematics teaching and learning.

Having established the overarching aim for your school, the teachers now break into subgroups to plan the actual study lessons.

In large schools these subgroups may include all teachers who teach a certain year/grade but in smaller schools, teachers from similar years might work together. For example, in elementary schools in Japan it is quite common that grade 1 and 2 teachers form one group, grade 3 and 4 teachers form a second group and grade 5 and 6 teachers form a third group.

From the work reported in Section 3, we would recommend where possible that:

- the group size should be four or five (but three would be possible).

If only two, there is no balance in views expressed; if six or more, there is not enough time for everyone to participate in the planning and review sessions and it would be difficult to release such a large number of teachers at one time.

At secondary level, we suggest using different classes in different year groups and that, for example, teachers of KS3 mathematics should have the chance to see GCSE or A-level classes. Similarly at primary, a mix of teachers in each group should ensure that participants see mathematics teaching in a variety of year groups, including Reception.

You might like to involve teaching assistants in the process as they will clearly have much to offer, although they may wish not to actually teach a lesson in the cycle of lessons to be followed.

You now have a focus for your lesson study and your groups have been chosen. Before you can start to plan lessons you need to establish the schedule of lessons and the choice of classes to be observed. This is probably not an issue at primary as each

participating teacher will probably be teaching mathematics to just one class. However, at secondary it is an issue and so, for the first cycle, it is probably best to choose classes that do not have substantial issues that might be problematic. These more challenging classes could be involved in a second cycle when staff have become more familiar with the process and have gained confidence with it.

Yoshida (**reference (11)**) suggests that teachers in Japan normally spend about 20 hours on one complete cycle. This includes planning, teaching the lesson, reviewing the lesson, re-teaching the lesson and writing up the final report. A considerable amount of this time is spent in planning the lesson and in particular in choosing a rich problem that will generate either multiple methods (i.e. the process is open) or multiple solutions (i.e. the solution is open). Further details of Japanese teaching methods are given in **Appendix 8**. As well as discussing the problem, Japanese teachers try to list all of the anticipated responses of their students. In this way they develop their own ability to see the lesson through the 'students' eyes'.

We recommend that specific time is designated for both this planning process and the review sessions. These sessions need at least 45 minutes as there must be enough time for an in-depth discussion (see below). It is important that these sessions are seen as important and should be free from interruption.

So we suggest that the leader:

- chooses the participants in each group (giving careful thought to the mix of the group, their experience, etc.)
- designs a schedule of sessions for each of the groups so that one cycle is covered in no more than one term (this of course has to be negotiated with the senior management so that any required cover, etc. is organised).

You are now ready for Stage 2!

Stage 2: Planning

From this point on, the groups work independently on their own cycle. The planning sessions are important meetings in which all participants should participate fully. This is essential to ensure joint ownership of the research lesson.

The planning process follows a four step process.

1. Identifying the topic
2. Mapping the unit of work
3. Identifying the lesson goals
4. Creating the lesson plan

Step 1 Identifying the topic

The group now meet to identify any areas that are causing concern. It is important that everyone openly shares their reasons for bringing these topics to the group. These might include topics that:

- have proved challenging for your students in the past;
- you personally find difficult to teach;
- are missing from your current scheme of work;
- could be taught in a more interactive way;
- fit into the agreed lesson study schedule.

Following discussion, the group then make a decision on the topic and the age group that will form the basis of this cycle. It is not necessary to identify who will teach the lesson at this point.

Our advice here is to:

- agree a topic of acknowledged importance to the group and document the reasons behind the choice.

Step 2 Mapping the unit of work

The lesson study process actually covers a whole unit of work which is now mapped out. The research lesson is identified by deciding which lesson will have the most impact on the unit. Through this mapping

process, the members of the team improve their own understanding of how the lessons will fit together and how the concepts will be developed.

Step 3 Identifying the lesson goals

The group now define the goals for both the unit and the research lesson. These will not only include both content and process goals but also goals relating to the long-term lesson study goals.

Step 4 Creating the lesson plan

The team now begins to work together to develop their combined plan for the research lesson. The collective ownership of the lesson is important and this is often maintained by initially not deciding who will teach the lesson although this is not always the case. Sometimes, two different teachers are identified who will both teach the lesson to different groups.

So we recommend that having identified the topic you:

- map out the unit into a series of lessons and then choose which lesson is key to the unit
- identify the learning goals for both the unit and the research lesson
- encourage everyone to think about the most effective way of teaching the lesson.

The lesson plan does not have to be totally original and the team are encouraged to share their own ideas and search for alternative approaches from a variety of sources. The aim is not to create the 'perfect lesson' but to create a lesson that 'makes the students' thinking visible' so that the group can learn more about how their students learn.

Fernandez and Yoshida (reference (17)) suggest that at this point the two teachers who are going to teach the lesson take over responsibility for completing the initial lesson plan. The two teachers meet several times over a short space of time and one is designated to write up the draft lesson plan although it is still seen very much as a joint project.

Fernandez and Yoshida also describe the lesson plan as a 'complex three part document' divided into sections.

- Section 1: Introduction to the lesson plan
- Section 2: Information about the unit
- Section 3: Information about the lesson

They go on to say the lesson plan describes the lesson 'blow by blow in a four column chart.'

Steps of the lesson: learning activities and key questions Time allocation	Student activities/ expected student reactions or responses	Teacher's response to student reactions/ things to remember	Goals and method(s) of evaluation

They further explain that the first column gives a description of the learning activities as well as the 'key questions (*hatsumon*) Appendix 8 that the teacher intends to ask at different points of the lesson'. As this column does include 'some verbatim lines for the teacher to deliver during key moments in the lesson', it has led to a misconception that lesson study means writing a rigid script (reference (8)).

Fernandez and Yoshida (reference (17)) state that the first column also indicates the different sections of the lesson; grasping the problem, presenting the problem format, solving the main problem, polishing and reporting individual solution methods, summary and announcement of the next lesson.

In the second column, the group describe the ideas, responses and reactions they expect to get from their pupils.

The third column not only outlines how the teacher will respond to the reactions expressed in column two but also lists important things the teacher should remember to do.

The fourth column serves as a 'running commentary about how the teachers will assess the success of different parts of the lesson'.

The draft lesson plan is then distributed to the other members of the group at least two days before the next group meeting.

So we recommend that:

- two teachers take responsibility for producing a draft lesson plan based on the joint decisions of the group.

At the next planning session, the two teachers who drafted the lesson plan, briefly describe the class and explain their reasons for the proposed activities. Each of the other group members should contribute their thoughts, suggestions, proposed revisions, etc., so that, by the end of the session, there is a lesson plan that all members of the group are in agreement with. This joint ownership is important as, at least in part, it takes away the feeling that the teacher is being assessed. It is the lesson plan that is the focus of review, the aim being to improve it for others to use in the future, after it has been observed, reviewed and refined.

Finally we suggest that:

- one teacher takes responsibility for producing the final lesson plan.

As stated above, this is a complex document which, initially, you are unlikely to be able to duplicate. It has taken considerable time to reach this point. A detailed sample lesson plan is given in **Appendix 3**.

You probably already have proformas for lesson plans, so when you first start lesson study, it is probably simpler to use your version. Or, you might like to use a tried and tested one, this is available in **Appendix 4**. A Word version is available at:

<http://www.cimt.plymouth.ac.uk/lessonstudy/>

This version can be used electronically.

You might also want to consider using established lesson plans that are freely available on the internet. Lesson plans developed by CIMT for Year 7 at:

<http://www.cimt.plymouth.ac.uk/projects/mepres/book7/book7.htm>

for Year 8 at:

<http://www.cimt.plymouth.ac.uk/projects/mepres/book8/book8.htm>

and for primary at:

<http://www.cimt.plymouth.ac.uk/projects/mepres/primary>

There are lesson plans for topical resources at:

<http://www.cimt.plymouth.ac.uk/resources/topical/>

Other sources of lesson plans for mathematics are given on the dedicated website:

<http://www.cimt.plymouth.ac.uk/lessonstudy/>

These lesson plans follow the Eastern European model rather than the Japanese model but form a good starting point for your team to get used to working collaboratively in planning lessons.

Stage 3: Teaching and observing the lesson

Even for experienced teachers, teaching in front of colleagues can be a stressful experience, so it is important that observers understand their role and follow a strict protocol.

They should arrive on time, be as cooperative as possible and ensure that the classroom ethos is positive. If the students are not used to being observed, they might react in a non typical way but it has been our experience that once the lesson starts (and even if it is being videoed) they soon settle down. It is good practice to involve the students early in the lesson study process by explaining to them its purpose and the role of the observers.

Unless there are good reasons to do otherwise, we suggest that the observers sit at the back of the classroom and, on the whole, do not participate in the lesson. This can be quite difficult, particularly when the students might look to them for help or comment.

Observers will want to see what the students are achieving in their written work so they do need to be able to move around the classroom when it seems appropriate. They must be sensitive to the position of the teacher taking the lesson and cause as little disruption as possible.

As the lesson plan represents the collective wisdom of the group, the teacher should, in general, follow the lesson plan closely. The observers will also have a copy of the lesson plan and, in particular, will be monitoring the student reactions and responses as outlined in columns 2 and 4.

Observers will also need to make notes on the lesson. It is very easy to forget even crucial events that happen in a lesson when you come to review it some time later. These can either be noted on the lesson plan or recorded separately.

The format you use for recording your observations can vary to suit your preferred style. In **Appendix 5** we have provided a proforma of an observation sheet which has been used extensively. This is also available, in both 45 and 60 minute formats, on the dedicated website.

At the close of the lesson the observers should thank the teacher for the lesson but not get involved in any discussion or evaluation. This should be left until the timetabled review session. If the lesson has been videoed, it is important for the group members to have a copy to reflect on, preferably before the review session. It would be too time-consuming to show the complete video (or even an edited version) at the review session.

In summary, the teacher should:

- include the students in the lesson study process so that they understand its purpose, their roles and the roles of the observers
- follow the agreed lesson plan.

The group members:

- should observe the lesson, taking notes of key points or incidents
- should not obstruct the flow of the lesson
- should not review or evaluate the lesson until the scheduled session.

Stage 4: Lesson review

It is important that this session takes place in a relaxed atmosphere and with sufficient time for an in-depth discussion. It would be helpful to have snacks and drinks readily available to reinforce the ethos as that of professionals working cooperatively and sharing ideas rather than making judgements on the effectiveness of the teacher! It can be helpful to hold the review in the classroom where the lesson took place.

All members of the group should attend this session as well as any 'knowledgeable others' that have been involved in the planning. The session should be chaired by one of the observers. The teacher who took the lesson should make the first contribution, reflecting on the lesson, indicating their feelings about it and explaining why they took any particular actions, especially if they moved away from the agreed lesson plan, and saying what they would change if they had the chance to give the lesson again.

The chair should then invite the other group members to give their reflections on the lesson, with each being asked initially to make one positive comment. They might then want to ask the teacher why they took specific actions and to consider what might have happened if they had made different decisions.

The team should then reflect on the lesson plan and specifically think about:

- the quality of the key questions;
- their ability to anticipate the students' responses to these questions (column 2);
- the timings of the lesson;
- the effectiveness of their evaluation (column 4);
- whether the goals of the lesson were met.

The last part of the review session is used to consider what action points follow from this exercise. In particular, the chair should obtain agreement from participants on revisions that should be made to the lesson plan so that an improved version is available for other staff in the school. There may be action points for the group which could influence the next lesson plan to be used or suggested action for the whole school.

“...any recommendations or issues for school/departmental policy resulting from a cycle of the group's work should be discussed at a full staff/departmental meeting.”

We have provided a proforma for this review session in **Appendix 6**, as well as on our dedicated website at:

<http://www.cimt.plymouth.ac.uk/lessonstudy/observing/>

In summary, we recommend that in the review session:

- one of the observers should act as chair
- the teacher should reflect on their lesson first
- each observer should have the chance to give their reflections (in a positive way) and to questions the teacher on action taken
- the chair should obtain agreement on revisions to the lesson plan and any other action points that are recommended by the group.

The review session might be difficult if the lesson has been particularly problematic! There are, however, usually at least some positive points that can be made, but nothing is achieved by pretending that everything was good and ignoring disasters. The chair must ensure that the discussion is conducted in a non-confrontational manner. It is better to consider why it went wrong and what actions might have averted the difficulties. It is also important to remember that the group has joint responsibility for the lesson plan and so any problems belong to them all. This could lead to interesting and constructive discussion which could well be beneficial to all the group members. It is crucial that amendments and changes are made to the plan before it is used again.

Stage 5: Future policy

Whilst the participants of each group will have gained much from the cycle of activity, there will be issues or recommendations for school/departmental policy that need to be taken further. So we suggest that:

- any recommendations or issues for school/departmental policy resulting from a cycle of the group's work should be discussed at a full staff/departmental meeting.

It is also important for this to be seen as a dynamic process and that one cycle of each group's work is just the start of the initiative. Each cycle should add to the group's understanding of how students learn and its effectiveness measured against the overarching aim.

To ensure that the process is continuous, we recommend that:

- the overarching aim should not be changed for at least three years.

It is also important that the dynamics within each group are sustained, and it is usually a good idea not to change the participants after each cycle. This ensures that the group get to know each other and build up the level of mutual trust necessary to maximise the potential of lesson study. To avoid polarisation of the groups it is important that where there is more than one lesson study group in a school, the groups periodically meet to share their findings. To ensure consistency, it is advisable to change the construction of the groups at the start of each school year.

So we recommend that:

- lesson study groups remain fixed for the whole school year
- lesson study groups periodically meet together to share their findings
- the leader should consider making changes to the membership of each group after each year of activity.

In **Appendix 7**, for easy access, we have summarised the protocols suggested above.

5. Sustainability of Lesson Study

“One of the dangers of lesson study is that it might result in sharing of poor practice.”

We have no doubt that a first and second cycle of activity will be enjoyed by all participants and will have benefits for both teaching staff and students. This is only a starting point though; lesson study is not a one off process but an underlying culture that can be used to:

- continually enhance the practice of teachers through professional sharing of experience
- induct new members of staff into a culture of classroom research and development
- improve the morale of staff, resulting in an improved retention rate (both in your school and in teaching).

To ensure that the full potential of lesson study is achieved, we will consider some issues which are important for sustainability.

Timetable

One of the key issues that all Pathfinder schools (Section 3) had to grapple with was the time required for this process. Undoubtedly, the optimum solution is to design the school timetable with lesson study integrated into the activities. This might involve regular use of supply cover, the use of classroom assistants for activities or using senior teachers to cover classes but whatever the strategy used, lesson study planning, observing and review sessions should be timetabled activities.

Although there might be associated costs, it should be noted that the benefits (see Section 6) are significant and the costs will be less than those incurred from sending staff on external CPD courses that will probably have little impact on classroom practice.

External catalyst

One of the dangers of lesson study is that it might result in sharing of poor practice. The choice of participants in each group should minimise this. An excellent way of making this less likely is to incorporate external consultants into the process to act as ‘knowledgeable others’. These people could be:

- local authority support staff in mathematics;
- teacher trainers in mathematics from your local university;
- expert mathematics teachers from other neighbouring schools (e.g. ASTs).

Although it might be difficult to secure the involvement of such people on a regular basis, their participation in even one lesson study (e.g. planning, observing, reviewing) has proved to be very effective in helping teachers to consider new strategies or approaches.

The external consultant acts as a catalyst for the process and should be invited not just as an extra observer, but to participate fully, taking a turn to teach a lesson. This is the basis of teacher training in University Practice Schools in many countries and is an important process for both the local authority support staff and teacher trainers to ensure that they do not lose their skills as teachers, whilst bringing new ideas and initiatives into your school.

Inter-school Lesson Study

Another complementary way of increasing the impact of lesson study is to join with neighbouring schools (in both primary and secondary sectors) for inter-school lesson study. There are some straightforward procedures to follow; for example,

- effective teachers should be observed
- visiting teachers should be given the lesson plans and information about the age and ability of the class
- on arrival, visiting teachers should be welcomed by the head or senior staff and be introduced to the teacher giving the lesson
- the teacher should impart information about the students and the work they have covered in recent lessons
- the OBSERVATION and REVIEW sessions should follow the lines of the protocols summarised in **Appendix 7**.

Careful planning is needed between the schools involved, but as an experience, this is

an excellent way of increasing the potential for teachers to share good practice.

Lesson Study Open House

This is the ultimate in lesson study and should only be attempted after fully experiencing within-school and inter-school lesson study.

It is used extensively in Japan and is a way of encouraging the public, and in particular parents, to understand how subjects, like mathematics, can be taught with flair and inspiration. They are so popular that many observers watch from the corridor or through the window. It is effectively a whole day event and often features multiple lesson study teams.

It is not always open to the public and may be restricted to invited educators.

As with inter-school lesson study, there are guidelines to be considered. For example,

- excellent teachers should be observed
- only lessons that have been reviewed, revised and shown to be effective are used
- initial information regarding the age and ability of the class is disseminated, together with the focus for the lesson study and the lesson plan

- a small team of experts, including teachers and external experts, are invited to take part in the REVIEW session
- invitations are given to the intended audience (for example, teachers from other schools, local authority staff, parents, etc.)
- the venue must be such that the class can be taught in classroom-type conditions and be observed by a large group of people (a lecture theatre, for example, would be ideal)
- the lesson is taught (about 45 minutes) and observed by all. After a short break, the lesson is reviewed by the team of experts with the teacher and the rest of the group following a strict timetable. This can take up to 90 minutes
- the Head thanks the teacher and experts and opens discussion with the observers (about 10 minutes).

Obviously these guidelines do not have to be followed exactly, but they do give an indication of the key activities needed to make an Open House lesson study a positive event for all concerned.

For Japanese teachers 'Open House' is the highlight of their year and it is often followed by a celebration in the evening.

6. Value of Lesson Study

“Develops personal skills in being able to give and take constructive criticism without causing or taking offence, rather seeing it as a means to professional learning.”

Having described some of the key procedures needed to implement lesson study, we will continue this manual by outlining some of the expected benefits to be gained through the use of lesson study for enhancing mathematics teaching and learning.

Catherine Lewis (**reference (10)**) states that the likely benefits of lesson study are:

- increased knowledge of subject matter
- increased knowledge of instruction
- increased ability to observe students
- stronger collegial networks
- stronger connection of daily practice to long term goals
- stronger motivation and sense of efficacy
- improved quality of available lesson plans.

Our experiences with lesson study indicate that this is indeed the case. To emphasise our experience we will illustrate some of these benefits through four detailed case studies. Based on these and the other schools that we have worked with, the points given below summarise the positive effects of lesson study CPD.

- Develops an awareness of what good teaching is.
- Develops reflective, critical analysis of teachers' own teaching and that of others.
- Raises teachers' awareness of different pedagogical strategies and develops their ability to evaluate these methods in a constructive way.
- Encourages all teachers in a department/school to collaborate in supporting and learning from each other and to regard problems as difficulties to be overcome, not as barriers to development.
- Develops personal skills in being able to give and take constructive criticism without causing or taking offence, rather seeing it as a means to professional learning.
- Encourages creativity, a willingness to take risks, try out new ideas and to share these experiences with colleagues.
- Quickly exposes to the whole department (or staff in a primary school) where further support or specific training is needed (e.g. planning, mathematics subject knowledge, classroom management, pedagogy, use of ICT, etc.) and places the responsibility for providing that support with the department (school). There might also be agreement that external expertise is needed to train the whole department (school).
- Allows teachers to become aware of what is being taught in earlier and later years and in what way, so that they understand the importance of their own role in their students' mathematical development.
- Encourages consistency of approach and standards across all years.
- Allows teachers to get to know students who are not in their own class and to become aware of their talents or problems.
- It is regular and ongoing, so that small improvements made in the quality of teaching and learning are embedded and built on.
- Creates a test bed and forum for trialling and evaluating new ideas, resources, teaching methods, equipment, etc.
- Involves teachers in their own professional development for the benefit of the whole school.
- Provides the opportunity for collaboration with other subject departments and schools as teachers can participate in other collaborative practice sessions beyond their own group of colleagues.

CASE STUDY 1:

Lesson Study at Grouville Primary School, Jersey

This is a large two-form entry primary school outside St Helier, with a popular and thriving Nursery and Reception. We have well-qualified keen staff and most classes have access to classroom assistants, many of whom are trained teachers.

When I became mathematics coordinator at the school, it was clear that we were not making as much progress in mathematics as might be expected and, with some trepidation and in-service support, as a group of staff we took the plunge and decided to use the MEP primary resources and teaching strategies to improve our mathematical attainment. This course is based on the Hungarian approach to teaching mathematics and provides a strong foundation in mathematics for children with really high expectations of what can be achieved. To help teaching staff (and that is essentially all our teachers as each one teaches their class) there are detailed lesson plans that can be adapted to suit our classes but we were under no illusions that this would be an easy option for many of our staff.

As well as having the detailed lesson plans and the teaching strategies well explained, the key to what has been a really successful implementation strategy has been the use of lesson study to both help us as professionals to share good practice and for the groups of teachers to share their concerns and find solutions appropriate to us.

We initially organised ourselves into four groups, each of four teachers, and included the Nursery and Reception teachers, with each group having a mix of teachers across the years. This has proved to be a valuable model to enhance our mathematics teaching. We are probably fortunate in that we do have a number of staff with a high mathematical understanding as well as being competent teachers and they have played an important role in the process, so much so that when we changed the groups round, they were the key members of each group.

Our initial concern about implementing the strategies required for MEP has transformed into excitement about what we are doing as a school. Staff now want to discuss mathematics teaching and more complex problems, both in mathematics and pedagogy, are a focus for such positive discussion. We are now nearing the end of the first year of our implementation and we have no doubts that we have already enhanced the mathematical progress of our children (and they are enjoying and responding to this new highly interactive style of teaching). So much so that we are ready to show teachers in other schools on the island and to encourage them to use lesson study if they go ahead with implementing MEP. A number of our teachers will be delighted to help our neighbouring schools by taking on the role of the outside expert teacher.

We are also convinced that this type of collaborative CPD will be great for other subjects in the school curriculum that we want to enhance. Lesson study provides an effective evaluative mechanism for taking on new approaches to the curriculum or teaching resources or teaching strategies.

Rachel Smith, *Mathematics Coordinator*

CASE STUDY 2:

Lesson Study at Princethorpe College, Rugby

As a new Head of Department, I found myself inheriting a group of very talented mathematicians who had perhaps lost a bit of their focus and who taught maths the way they always had without really thinking about different approaches.

Although each member of the department went on an external INSET course last year, they didn't really help as they weren't applicable to Princethorpe College and more time was spent travelling than training. The lesson study model for CPD appeared to redress this balance.

We split into groups of four staff and we are teaching one ourselves and observing the other three in this academic year. As HoD I went first at teaching. It was obvious from the first two minutes of the first planning session just how much of a positive impact this was going to have. Immediately we started talking about how to teach sequences to Year 7 there were comments such as 'I had never thought of that but what a good idea', 'If you were to do that but do it this way...'. We realised that although we talk about school policies, strategies, and pupils we had never really sat and talked about our teaching. It was very refreshing and stimulating to talk about our passion rather than other things associated with it.

The observations are useful as they allow you to watch different delivery methods and different styles from which you can take any points that you think may improve your teaching and adopt them. More than this, being in colleagues' classrooms has brought a much more team feeling to the dept, we know more about each other as teachers. It has been wonderful for struggling colleagues, say with discipline, to be able to watch and be watched without it being specifically as a measure to help them improve their discipline. Also part-time members of the department take an equal part in this so they have also felt much more involved.

The lesson reviews went very well and were very positive. Any criticisms were at the lesson plan, not the teaching, and so were acceptable to all. Again a lot of discussion was stimulated and sharing of good practice was apparent. The amazing thing to me was colleagues who had been quite sceptical were volunteering to go next and suggesting focuses for their lessons – e.g. 'Could we plan a lesson to help me try different styles of questioning?'.
'

It has without doubt brought us closer as a department. Already our teaching methods are improving as we share good practices. People are more willing to share experiences as well now and in the staff room it is not unusual to discuss how one colleague has used something suggested at a planning meeting in another lesson with success.

SMT are very impressed with how this is going and consequently have been very supportive. I have twice given presentations on this method of CPD, to a HoD meeting and a full staff meeting respectively. Other departments are now taking up this method of CPD. I am even taking it further and myself and the maths teachers at our associated junior school are going to plan observe and review each other as well.

The Head would also say that the money saved by not going on external INSET is a big advantage as well!!!

Mike Conroy-Hargreaves, Head of Mathematics

CASE STUDY 3:

Lesson Study at Roseland Community School, Cornwall

This is a small 11–16 community school in a rural location with a good record in GCSE mathematics. I have been teaching now for ten years and have taught in seven schools, as a normal teacher in a department. It was very clear to me that all teachers, especially, mathematics teachers, tended to work on their own.

When I became HoD at a previous school, I tried with varied success to get the staff to work together, working in pairs, working on projects or even spending time on discussions on how you teach certain areas e.g. solving equations. This was on my own initiative with no external help and no guidance. It was hard and had only limited success.

When I joined the Roseland the department was used to spending one hour a week together; previously this had been used for basic paperwork, and also an attempt had been made at some joint practice. On discussion with individuals concerned it had mixed success, not everyone felt involved. Through my previous experiences I know how difficult it is but as a Pathfinder school, I no longer felt on my own trying to make it up as I went along. I now have others to turn to, to email with questions, and someone who is an expert in this field and knows what people have tried and who can guide me and the department; there is someone outside the department with an external view.

The advice I was given was to get everyone involved. So now no one is left out and we all feel involved and have input. It's a simple device really: we all agree to teach the lesson we plan (so everyone necessarily feels they have to input) and then once we have agreed on the lesson, throw a dice to see who teaches it. The initial planning with no one knowing who will teach it and the random throwing and thus random choice of deliverer of the lesson works like a dream to get everyone involved and I would never have thought of it in a million years.

The observation sheets allow us to effectively see an overview of the lesson and the feedback session where the person observed speaks first and all have input, is covering a range of pedagogical areas that will improve the teaching of all us in the department and to be honest it is already.

The model has legs! So far, the comments have been so positive; for example, from the NQT 'Doesn't every mathematics department do this? It's so good.'; the experienced teacher has said how fully involved she feels and how this is inspiring her; the Assistant Head has said that doing the planning sessions when he can is the most enjoyment he gets – just talking about teaching and how to do it. Everyone involved wants it to carry on, since we all know it's the most effective and enjoyable way to improve our practice, and make it specific to our needs.

Sean Walker, Head of Mathematics

CASE STUDY 4:

Lesson Study at Bishop Luffa CE School, Chichester

This school is a successful, oversubscribed voluntary aided Church of England mixed comprehensive school of 1392 pupils.

The Mathematics Faculty is well regarded in the area for both its outstanding examination results and its innovative approach to teaching and learning. It consists of ten full time specialist teachers with varying backgrounds and different levels of experience. It is led by the Head of Faculty and the Faculty Coach.

Over the past 12 years, our Faculty has been involved in various forms of collaborative practice and has a culture of working together on 'good lessons that work'. During the last five years, the Faculty had been working with the Centre for Innovation in Mathematics Teaching (CIMT) as one of their 'Pathfinder Schools'. The Pathfinder schools project was intended to be a test bed for a new collaborative practice model for CPD in which teachers continually worked together to improve their understanding of effective teaching and learning. It involved meeting together regularly to observe, analyze, discuss and reflect on each other's mathematics lessons in a spirit of cooperation and mutual support. Although our head teacher was supportive of the concept, he was unwilling for us to be released from our normal timetables so the meetings initially took place after normal hours. Likewise we were unable to physically watch lessons through timetabling constraints but we were offered the help of Mike Hindle from CIMT who not only advised us on our teaching and learning approaches but also agreed to film the lessons. Initially, the department was totally against the idea as they felt they were already incredibly busy but they agreed to meet Mike who explained that this was not another top down initiative but a chance for the department to shape their own CPD programme to match their own needs. He described this as a 'bottom up' approach where everyone is seen to be of equal importance and everyone has something to offer the others.

For this project, the Faculty divided into two teams; one led by me as Head of the Faculty and the other by the Faculty Coach.

Our initial focus was on the technical aspects of our teaching such as the balance of teacher/pupil talk. In particular, were we asking the right sort of questions to provoke in depth discussion between the pupils? Were we then giving the pupils adequate time to discuss these questions? Were we giving them enough time to explain their thinking to the rest of the group? We soon realised that we needed to plan jointly the lesson with this in mind rather than use 'ordinary' lessons and so we began to move towards lesson study. At this point we didn't really understand the rationale underpinning Japanese lesson study and were still concentrating on teaching techniques rather than focusing on an overarching aim for pupil change. Our understanding of lesson study at this point was simply one of sharing good practice.

We tried to complete a cycle every four weeks but, after one term of frenetic activity, we realised this was impossible to sustain. Nevertheless we all learned a great deal from each other and everyone enjoyed the experience with meetings often going on for two to three hours, simply because no one wanted them to end. In the second term we tried to complete a cycle every half term but, eventually, as we began to understand more about the lesson study, even this was too many.

Things really started changing when I joined Mike and 12 other teachers on a trip to Japan to see lesson study in action. We visited three different schools and in each were allowed to observe

both the actual lesson and the debriefing session. Imagine our surprise when, as well as us, over 50 teachers packed into the classroom to watch the first lesson. It was simply awesome and left us all wanting to know more about lesson study and what we had actually been privileged to see.

I had read 'The Teaching Gap' (**reference (4)**) and so was aware of the importance of cultural scripts in teaching but I still was overwhelmed by how different Japanese lessons were to my own. On our return, Mike and I made our mission to find out as much as we could about lesson study. Many of our findings are included in this report.

The first change at school was to explain the logic behind having an overarching aim for our students and convincing my team that we actually needed one if we were to move on to the next level of lesson study. After a whole faculty discussion, we initially settled for 'Our students should enjoy learning mathematics.' From this moment on, all of our planning centred on two questions. 'What do I want to my students to learn from the lesson?' and 'How do I make this learning enjoyable?' This meant that in each cycle, not only did we focus on the mathematics involved and improving our teaching skills but we began to think more about seeing the lesson through the students' eyes.

Without doubt over the next two years all of us improved as teachers and our students attitude towards mathematics changed so much that not only have our results improved significantly but the number wanting to study A level in the sixth form has risen from an average of 30 per year to over 90 last year. There have been other consequences; the head teacher has become much more supportive and we now have curriculum time allocated for lesson study; the rest of the school have become interested and three other faculties now use lesson study as their preferred method of delivering CPD; gradually lesson study is beginning to spread across West Sussex as members of my team frequently visit other schools in the area to spread the word and this has led to various members of my team gaining posts of responsibility in other schools.

Although over this period of time there have been several changes of personnel, the leadership of the faculty has remained unchanged and has the trust of the other members. The structure of the two lesson study teams has changed over the last three years due to timetabling constraints but the leadership has remained constant.

In the last academic year we revisited our overarching aim*. We have also begun to base our teaching on the Japanese model of structured problem solving that so impressed me on my visit to Japan**. Each of our groups now completes two cycles per year.

Over this period of time, all of us in the Mathematics Faculty have been reinvigorated and our enjoyment of teaching has grown tremendously. Without doubt, we all feel that lesson study has been the catalyst for this growth.

Derek Robinson, Head of Mathematics

* This is described in Section 7 of this manual

** This is explained in **Appendix 8**

7. Identifying the Research Theme or Overarching Aim

“Our students will become independent thinkers (learners) who enjoy working together to produce creative solutions in unfamiliar situations.”

As stated earlier, lesson study teams use one of three methods to determine their overarching aim: identifying gaps between ideal and actual student qualities, building on existing school improvement goals or using school mission statements. In this section we follow the progress of one of our Pathfinder schools as they set about creating their ‘overarching aim’.

Rather than creating yet another list, we eventually decided to create our aim by looking at the School’s mission statement. Although the mission statement was very broad and lengthy we decided to focus on the section headed, ‘To foster in pupils an enthusiasm for learning and discovery’, as this seemed to fit our own views on how pupils learn most effectively. We also decided to link this with our on-going work on ‘Providing All Children with the Foundations for Achievement’ Bernard (**reference (23)**) with its emphasis on developing students ‘Confidence, Persistence, Organisation and Getting Along’ or ‘CPOG’ for short.

We met together on an inset day, projected the relevant section of the mission statement on the whiteboard, talked it over in pairs and then wrote our thoughts on the whiteboard. In this way we hoped to make the mission statement mean more to us and come alive for the pupils in their lessons.

In pairs we then tried to reduce this to one single sentence which would eventually become our research theme or overarching goal. In true lesson study style, each pair presented its own ideas to the rest of the group for consideration. After much ‘polishing’ of these ideas we eventually settled on:

‘Our students will become independent thinkers (learners) who enjoy working together to produce creative solutions in unfamiliar situations.’

We then turned the overarching aim into specific, measurable objectives that we would eventually use to evaluate the success of our research lessons over the next three years.

Objectives

- *Enjoy doing mathematics* – to help students learn to enjoy and sense personal reward in the process of thinking, searching for patterns and solving problems.
- *Gain confidence and belief in their abilities* – to develop students’ confidence in their ability to do mathematics and to confront unfamiliar tasks.
- *Be willing to take risks and to persevere* – to improve students’ willingness to attempt unfamiliar problems and to develop perseverance in solving problems without being discouraged by initial setbacks.
- *Interact with others to develop new ideas* – to encourage students to share ideas and results, compare and evaluate strategies, challenge results, determine the validity of answers and negotiate ideas on which they all can agree.

At this point we had effectively identified our desired outcomes for the next three years. We now knew where we were going but how would we know when we got there?

We then discussed how we would know if our students had actually achieved these results. In other words, what sort of evidence would we accept as validation that the desired learning had been achieved? This was a real eye opener! It had been relatively easy to come up with an impressive list of objectives but it was much more of a challenge to decide on how these objectives could be assessed over a long period of time. In lesson study you learn as you go along.

This discussion naturally led us into another discussion as to what sort of activities were most likely to develop the qualities that we had identified above. This debate is also ongoing but we did decide to look at the Japanese model of teaching mathematics.

Details of the Japanese Approach can be found in **Appendix 8**.

8. Final Thoughts

“ We hope that you enjoy the experience of lesson study. It is the most effective form of CPD that we have experienced! ”

James Stigler (**reference (24)**) summarised his findings from various video research studies of mathematics teaching by saying that, ‘The key to long-term improvement [in teaching] is to figure out how to generate, accumulate, and share professional knowledge.’

Isoda *et al.* (**reference (25)**) stress that is exactly what lesson study does in Japan.

Our challenge is to make that happen in the United Kingdom.

We hope that this manual will help you and your staff to implement some form of lesson study and be part of this process. We have given these guidelines to help you to implement lesson study in your school, minimising the pitfalls and maximising the benefits. All the information, proformas and procedures in this manual are here to help you, not to straitjacket the way you use lesson study. You need to be professional in how you adapt, revise and use the resources.

You can also find more help at our dedicated website:

<http://cimt.plymouth.ac.uk/lessonstudy/>

We are not trying to say that lesson study should be the only form of CPD used in your school (department) but we are saying that important and effective lesson study can enhance mathematics teaching and learning in your school (department) whilst helping your teachers to share their practice and grow professionally.

Throughout this manual we have tried to emphasise that lesson study can't be learnt by reading about it. It can only be experienced through participation. As Antonio Machado one said, ‘*Traveler, there are no roads. The road is created as we walk it together*’ (**reference (26)**).

Certainly our understanding of lesson study has grown during the last five years and we now see it very much ‘as a means of generating, accumulating, and sharing professional knowledge’.

We hope that you enjoy the experience of lesson study. It is the most effective form of CPD that we have experienced!

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Appendix 1: EPPI Review of Collaborative Continuing Professional Development

Introduction

'Collaborative CPD' refers to programmes where there were specific plans to encourage and enable shared learning and support between at least two teacher colleagues on a sustained basis. 'Sustained CPD' refers to programmes that were designed to continue for at least twelve weeks or one term. In 14 of the 15 studies synthesised by one review, collaborative CPD was linked with improvements in both teaching and learning; many of these improvements were substantial.

Impacts on teachers

They showed greater confidence and had enhanced beliefs in their power to make a difference to their students' learning (self efficacy).

They developed enthusiasm for collaborative working, notwithstanding initial anxieties about being observed and receiving feedback.

Collaborative CPD was embedded in many cases in the development of collaborative practice such as joint planning and team teaching.

Teachers showed a greater commitment to changing practice and willingness to try new things. For example, they made use of specific tools or interventions which introduced greater collaboration relating both to generic learning processes, such as activities to generate more effective and targeted dialogue between students, and to specific teacher activities, including, for example:

- a conscious effort by teachers to use computers more for both instruction and collaborative planning; or
- a conscious effort to increase the range of teaching and learning strategies targeted at specific student needs.

Positive outcomes of the impact of collaborative CPD sometimes emerged only

after periods of relative discomfort in trying out new approaches; things often got worse before they got better. Collaboration was important in sustaining change.

Time for discussion, planning and feedback, and access to suitable resources, were a common concern in many of the studies reviewed.

In comparison, in studies of individually-oriented sustained CPD, two found some impact on teachers' practice and beliefs respectively, and one found a minimal effect on teachers' efficacy. The evidence was considered to be weak because not enough studies were found.

Impacts on students

Students showed enhanced motivation and confidence, increased participation, and increased satisfaction with their work.

- They showed more positive responses to specific subjects.
- They showed improvements in learning, and in performance, such as improved test results, greater ability in decoding and enhanced reading fluency.
- They demonstrated better organisation of their work.
- They showed increased sophistication in response to questions.
- They experienced a wider range of learning activities and strategies.
- There was some evidence of positive effects on students' behaviour.
- There was some evidence that collaboration among teachers acted as a model for collaboration among students.

In comparison, studies of individually-oriented sustained CPD showed modest impacts focused on behaviours and attitudes rather than learning outcomes, which were not measured.

CPD processes

Core features of CPD processes which were linked, in combination, to positive outcomes include:

- The use of external expertise linked to school-based activity.
- Feedback (usually based on observation).
- Being based in the learning teacher's classroom.
- Involving the teachers in applying and refining new knowledge and skills and experimenting with ways of integrating them in their day-to-day practice.
- An emphasis on peer support rather than leadership by supervisors.
- Scope for teacher participants to identify their own CPD focus, starting points and pace.
- Processes to encourage, extend and structure professional dialogue as well as ongoing collaborative working.
- Processes for sustaining the CPD over time to enable teachers to embed the practices in their own classroom settings. However, there was no clear link between impact and length of time beyond 12 weeks.
- Working in pairs or small groups, which may be more effective than larger discussion groups.

Specialist input

Positive impacts found in specialist-provided CPD were:

- Specialists built the CPD processes on what teachers knew and could do already, with an emphasis on individual learning.
- In most cases, the CPD lasted longer than two terms, and the specialist contact with teachers (both scheduled and 'on call' sessions) took place over 10 days or more.
- Specialists encouraged and guided the teachers in supporting each other.
- Specialists introduced the theoretical and practical knowledge base.
- Ongoing specialist support included modelling, workshops, observation and feedback, coaching, and planned and informal meetings for discussion.

Appendix 2: Possible Focus for Lesson Study in Mathematics

• Lesson preparation
• Resources and activities
• Use of IT (e.g. PowerPoint, other software, spreadsheets)
• Seating
• Students on task
• Students demonstrating in front of the class
• Paired work
• Teacher praise
• Explanations
• Contexts and applications
• Functional mathematics
• Questioning techniques
• Reviewing answers
• Challenge and extension work
• Lower attaining students
• Notation, layout, language and precision
• Pace of activities
• Use of teaching assistants
• Homework
• Mathematics subject knowledge
• Monitoring of student progress
• Time management
• Assessment

Appendix 3: A Sample Lesson Plan

This is the lesson plan produced by a group of five teachers from Bishop Luffa School in February 2009. It is an example of their current practice.

Date:	Tuesday, February 10th 2009
Group:	8L2
Period and Location:	P2 R38
Teacher:	E Arnold
Lesson study group:	D J Robinson, S. Charge, E. Arnold, S. Hine, G. Bennett

I. Background information

Goal of the Lesson Study Group:

Most of the pupils who arrive at Bishop Luffa are cheerful and enthusiastic about school although some may have little confidence in their own mathematical ability following their experience at KS2. It is because of this and the fact that some pupils become discouraged as they progress through the school that our overarching aim is that *'Our students will become independent thinkers (learners) who enjoy working together to produce creative solutions in unfamiliar situations.'*

This means that lessons should be both enjoyable and structured. There should be clear learning objectives that are shared with the pupils as the lesson unfolds. These may or may not be revealed at the beginning of the lesson. To ensure learning has taken place pupils should be given the opportunity to express their understanding of these key objectives in their own words during the lesson.

We believe that, in general, pupils intrinsically enjoy tackling puzzles and solving problems that are sufficiently challenging as to make them worth solving but not too difficult as to make them inaccessible to the majority of the group. Consequently, in most lessons, there should be clear evidence of this 'problem solving' culture.

Our research theme (the overarching goal):

'Our students will become independent thinkers (learners) who enjoy working together to produce creative solutions in unfamiliar situations.'

Objectives:

- *Enjoy doing mathematics* – to help students learn to enjoy and sense personal reward in the process of thinking, searching for patterns and solving problems.
- *Gain confidence and belief in their abilities* – to develop students' confidence in their ability to do mathematics and to confront unfamiliar tasks.
- *Be willing to take risks and to persevere* – to improve students' willingness to attempt unfamiliar problems and to develop perseverance in solving problems without being discouraged by initial setbacks.
- *Interact with others to develop new ideas* – to encourage students to share ideas and results, compare and evaluate strategies, challenge results, determine the validity of answers and negotiate ideas on which they all can agree.

Notes:

Structured problem solving

Shimizu (2007) suggests that in all Japanese schools there is clarity about both the importance of teaching through problem solving and the most effective way of doing so. He

describes these lessons as 'structured problem solving' which follow a sequence of five phases:

- reviewing the previous lesson;
- presenting the problem of the day;
- students working individually or in groups;
- discussing solution methods; and
- highlighting and summarising the main point.

This methodology for problem solving is described in more detail in **Appendix 8**.

Background Information

This is Emma Arnold's group (set 1 out of 5).

Emma identified 5 normally quiet students and provided a seating plan.

II. Unit Information

Name of the unit:

Probability (Unit 10 MEP 8A) (5 lessons)

Goal(s) of the unit:

In this unit we extend the ideas first met in Year 7 where students studied single event probability to now include 2 or more events.

They will be encouraged to develop their own systematic methods of recording all possible mutually exclusive outcomes for two successive events.

They will compare experimental and theoretical probabilities, including those based on equally likely outcomes and explain their findings using the correct language.

Students will use relative frequency as an estimate of probability and use this to compare outcomes of experiments.

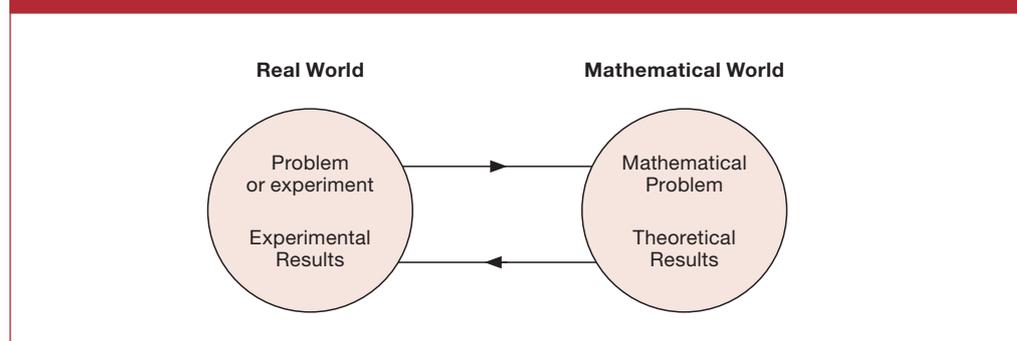
As well as extending students' understanding to 2 event probability, the underlying theme is one of matching theoretical probabilities or frequencies with experimental values.

Essentially this is the concept of mathematical modelling, summarised below in Figure 1.

In addition, this unit will:

- encourage pupils to think mathematically about real world problems.
- use problem solving to enable pupils to construct their own mathematics.
- stimulate pupils' interest and desire in learning mathematics.
- continue to build an atmosphere of mutual respect in the classroom where ideas can be freely exchanged.

Figure 1: Mathematical modelling



How this unit is related to the curriculum:

NC Programme of Study:		
The study of mathematics should include: Experimental and theoretical probabilities, including those based on equally likely outcomes.		
Previously learned concepts	Year 8 S.O.W.	Concepts to be learned in the future
Yr 7 Unit 21 (7B)	Yr 8 Unit 10 (8A)	Yr 9 Unit 6 (9A)
<p>NS – learning objectives</p> <p>Use vocabulary and ideas of probability, drawing on experience</p> <p>Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts; identify all the possible mutually exclusive outcomes of a single event</p> <p>Estimate probabilities by collecting data from a simple experiment and recording it in a frequency table; compare experimental and theoretical probabilities in simple contexts</p> <p>NC Attainment targets:</p> <p>Level 5 They understand and use the probability scale from 0 to 1.</p> <p>They find and justify probabilities and approximations to these by selecting and using methods based on equally likely outcomes and experimental evidence, as appropriate.</p> <p>They understand that different outcomes may result from repeating an experiment.</p> <p>Key language</p> <p>Use, read and write, spelling correctly:</p> <p><i>fair, unfair, likely, unlikely, equally likely, certain, uncertain, probable, possible, impossible, chance, good chance, poor chance, no chance, fifty-fifty chance, even chance, likelihood, probability, risk, doubt, random, outcome...</i></p> <p>Use vocabulary and ideas of probability, drawing on experience.</p>	<p>NS – learning objectives</p> <p>Interpret the results of an experiment using the language of probability; appreciate that random processes are unpredictable</p> <p>Know that if the probability of an event occurring is p then the probability of it not occurring is $1 - p$;</p> <p>Use diagrams and tables to record in a systematic way all possible mutually exclusive outcomes for single events and for two successive events</p> <p>Compare estimated experimental probabilities with theoretical probabilities, recognising that:</p> <ul style="list-style-type: none"> • if an experiment is repeated the outcome may, and usually will, be different • increasing the number of times an experiment is repeated generally leads to better estimates of probability <p>NC Attainment targets:</p> <p>Level 6 When dealing with a combination of two experiments, they identify all the outcomes.</p> <p>When solving problems, they use their knowledge that the total probability of all the mutually exclusive outcomes of an experiment is 1.</p> <p>Level 7 They understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.</p> <p>Key Language</p> <p>Use vocabulary from previous year and extend to:</p> <p><i>event, theory, sample, sample space, biased...</i></p> <p>Use the vocabulary of probability when interpreting the results of an experiment; appreciate that random processes are unpredictable.</p>	<p>NS – learning objectives</p> <p>Interpret results involving uncertainty and prediction</p> <p>Identify all the mutually exclusive outcomes of an experiment; know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving problems</p> <p>Compare experimental and theoretical probabilities in a range of contexts; appreciate the difference between mathematical explanation and experimental evidence</p> <p>NC Attainment targets:</p> <p>Level 8 They understand how to calculate the probability of a compound event and use this in solving problems.</p> <p>Key Language</p> <p>Use vocabulary from previous years and extend to:</p> <p><i>exhaustive, independent, mutually exclusive, relative frequency, limit, tree diagram...</i></p> <p><i>and the notation $p(n)$ for the probability of event n.</i></p> <p>Use the vocabulary of probability in interpreting results involving uncertainty and prediction.</p>

Instructional sequence for the unit:

Lesson 1 – This will recap one event probability using 4 questions from the PDS pack on probability.

Lesson 2 – (research lesson) – This will focus on comparing experimental and theoretical probabilities in a problem solving context.

Lesson 3 – This will focus on probability using listing methods (sample space).

Lesson 4 – This will focus on probability using tree diagrams.

Lesson 5 – This will focus on conditional probability.

III. Research Lesson Information

Name of the study lesson:

Two yellows (**Lesson 2**)

Goal(s) of the study lesson:

The aims of this lesson are to:

- extend pupils' understanding of probability by introducing them to two event probability through problem solving.
- compare their initial thoughts ('gut feeling') with their experimental probability results.
- devise their theoretical model by systematically recording all outcomes.
- encourage pupils to develop their own methods for solving problems.
- give pupils a chance to apply their knowledge to solve a real world problem.

How this study lesson is related to the lesson study goal and objectives:

This lesson will begin by reviewing the homework which involved a recap of one event probability. Pupils will then be given an unfamiliar problem involving two spinners.

[Be willing to take risks and to persevere]

They will be asked for their 'gut reactions' and then asked how these first reactions could be tested.

They will almost certainly suggest both experimental probability and listing all outcomes (theoretical model).

They will be given spinners and asked to predict what will happen.

[Enjoy doing mathematics]

They will then test their findings and compare results.

They will then be asked to come up with their own method for recording all of the outcomes.

[Gain confidence and belief in their abilities]

They will share their ideas and reach an agreement on which is the most effective method offered.

[Interact with others to develop new ideas]

They will then compare their experimental results with the theoretical model.

Although the lesson is fairly informal, it will give the pupils a chance to develop their own understanding about both experimental and theoretical probability. There will be a balance of auditory, verbal and kinesthetic activities with a major emphasis on pupils working together and pupils working at the main board. Pupils will be asked to explore an unfamiliar problem and share their ideas. Wherever possible, they will share their own answers with their partner before sharing more generally.

Agreed Lesson Plan:

Steps of the lesson: learning activities and key questions (and time allocation)	Student activities/expected student reactions or responses	Teacher's response to student reactions/ Things to remember	Goals and Method(s) of evaluation
Explanation of why this lesson is being filmed and why all these people are here.	Worried faces.	Reassure.	Were students reassured? (Check with questionnaire)
Review of the homework.	Students up to the board.	Encourage, reflect questions back to the class.	Did all students understand the homework?
Research context.		Make sure students are in pairs. If an odd number then one person will have to have two spinners.	
Question explained. Put it on the whiteboard. What is the probability that both spinners point at yellow?	Students read the question.	It will also be on their sheet.	Do all the students understand the question?
Without talking to anyone else think about it carefully. Do you agree with Paul? Do you agree with Julie? Perhaps you have a different idea of your own? Maybe you are not sure, like John.	Students think about it.	Teacher encourages them if necessary to think about their own ideas.	Where engaged with the question? Do they appear to find it interesting?
What is your first reaction? Write your answer on the whiteboard and show. Write down their initial thoughts on their sheet.	Students write down their initial thoughts with reasons on their research lesson sheet.	Teacher records answers on the board. (Keep for later.)	Did all students answer? Did the students respond enthusiastically? Did the students offer a variety of answers?
Now share your thoughts with your partner. Convince them that you are right!	Students share their ideas.		Do students share their ideas? Do they listen to each other? Did the students explain their reasons clearly?
Has anybody changed their mind?	A few hands go up?	Teacher asks for their new answers. Add to the tally chart (underneath).	Were the students open to other points of view? Did any of them change their minds?
What do you think John meant? (on their research sheet)	We could make a table. We could actually use spinners to find out.	We certainly could. Can you think of another way? Funny you should say that because we have actually made some spinners for you to use today.	Did the students explain their ideas clearly?
Give out the spinners – one per student.	Students excited.		Were the students enthusiastic about using spinners?

Steps of the lesson: learning activities and key questions (and time allocation)	Student activities/expected student reactions or responses	Teacher's response to student reactions/ Things to remember	Goals and Method(s) of evaluation
I want both of you to spin your spinner anti-clockwise once.	Students spin their spinners	Make sure students put spinners flat on table and spin anti-clockwise	Did all the students understand what they had to do?
Does it matter what order you write down your results?	Yes No	Respond positively for both answers. Leave as a 'maybe'	
Can one of you write down on your board what happened? I think we will put the person on the left result's first – just in case that is important	Students write their answers on the whiteboard Show when asked	Make sure students write down 2 colours	Students show answers on white boards. Did all students answer? What answers were there?
If you spin your spinners 20 times each, how many times do you expect to get the 2 yellows?	Students write down their answers on sheet and on white board. Show when asked	Teacher records their predictions on the board.	Did the students use their earlier ideas on probability to estimate their results?
How can we record all of your results systematically?	Write them down Make a table	Teacher listens encouragingly, thanks each person for their suggestion – non judgmentally	Did they students understand that they only needed to record success and failure?
Talk to your partner and decide how you are going to do this	Students talk to their partner	Teacher circulates Makes notes on possible ideas	Did students talk to each other? Did they reach some sort of consensus?
Now I want you to actually spin your spinners and record the results systematically.	Students spin away and record their results	Teacher circulates, encouraging	Did students record their results systematically? Are they recording all outcomes or simply success and failure?
How many times did you get 2 yellows? Write your answers on the white board.	Students write down their answers. Show boards.	Teacher scans results and records on the board	Do students all tell the truth or do they want to 'win'?
Talk over with your partner whether you think your results are what you expected	Students talk over their results.	Teacher circulates listening in and encouraging.	Did the students talk enthusiastically about their results?
Teacher looks at the results on the board 'Can that be right? You didn't all get the same?' Talk it over.	Students look at everyone's results. You wouldn't expect them to be the same because you get different answers every time you spin the spinner.	Teacher circulates listening in and encouraging.	Did students understand that experimental probability will not always give the same results?
Can you explain to me why this happened?	Students' hands go up.	Teachers asks one of the quiet students	Did he/she answer confidently
How can we get a better result?	Have more goes Put our results together	Teachers listens encouraging, thanks each person for their suggestion – non judgmentally.	Do students understand the significance of the law of large numbers?

Steps of the lesson: learning activities and key questions (and time allocation)	Student activities/expected student reactions or responses	Teacher's response to student reactions/ Things to remember	Goals and Method(s) of evaluation
Teacher asks students to add up the scores so far. Write down the answer on the sheet.	Students add up scores. Write down the answer on the sheet.		
How do we work out the probability of getting two yellows from these results?	Count up how many times we got 2 yellows and count up how many goes we had.		Do students appear to understand relative frequency?
First key point: <i>I can use relative frequency as an estimate of probability and use this to compare outcomes of experiments.</i>			Do all students understand the first key point?
According to our results, we have got 2 yellows ??? times out of ??? times. So what is the probability of getting 2 yellows?	Students work out their answer as a fraction or a decimal.		Did the students know what calculation to carry out?
How does this compare with your original ideas? Talk it over with your partner.	Students check back and talk over their results with their partner		Did students talk to each other? Did they reach some sort of consensus?
Are you convinced this is right answer?	Yes No	Why? Why?	Are some students still unsure because it is experimental probability? Are some students convinced because we have a large number of spins now?
So far we have been using experimental probability to find an answer to our question	Students listen		Do the students appear to understand the concept of experimental probability?
We can also use theoretical probability to work out what should happen in theory. This called mathematical modelling. In other words we don't have to actually do the experiment, we can work out what should happen.			Did the teacher make the difference between experimental probability and theoretical probability clear?
If we wrote down all possible results, how many different results could we get? Can you think of a way of recording all the different possible results? Talk it over	Students talk it over Come up with suggestions	Teacher circulates, noting who has done what. Teacher orders responses in her mind ready to call up some students to the board.	Did the students come up with a variety of approaches?

Steps of the lesson: learning activities and key questions (and time allocation)	Student activities/expected student reactions or responses	Teacher's response to student reactions/ Things to remember	Goals and Method(s) of evaluation
Teacher calls up students to the board in order. Gets them to share their ideas.	Students come to the board in turns.	Teachers listens encouraging, thanks each person for their suggestion – non judgmentally.	Did the teacher call up the students in a logical order?
Polishing the students' answers to get to where you want to get (known as 'Neriage' in Japan).	Students compare different methods.	Teacher encourages them by asking provocative questions.	Did the teacher manage to get the result she wants without the students realising?
Agree on the best way forward. For two events we can use a two-way table.	All students now write down the two-way table on their sheets.	Make sure we now all follow the same system – which should be derived from their solutions.	Were all students able to construct the two way table?
This type of mathematical modelling is called using a sample space diagram.	Students listen.		Were all students listening attentively? Were some still playing with the spinners?
Could you explain to your partner what we mean by a sample space diagram?	A sample space is just a systematic way of recording all of the possible outcomes where the order matters.	Teacher circulates and listens in.	Can students explain what is meant by a sample space?
Second key point: <i>I can use diagrams and tables to record in a systematic way all possible mutually exclusive outcomes for single events and for two successive events</i>			Do all students understand the second key point?
So based on our sample space, how many outcomes are possible? Write it on your white board.	10 16	Why do you think there are 10? Why do you think there are 16?	Does anyone put 10? Can students explain why 10 might be the answer?
So based on our sample space, what is the probability of getting 2 yellows? Write it down on your white board.	1/16		Did everyone get the right answer?
How does this compare with the experimental probability?	Students compare the two answers?		Were students able to compare the two answers?
Are you surprised by this? Talk it over with your partner.		Experimental probability usually gives a good estimate of the probability if you have enough goes.	Were the students surprised? Did they talk it over with their partners?

Steps of the lesson: learning activities and key questions (and time allocation)	Student activities/expected student reactions or responses	Teacher's response to student reactions/ Things to remember	Goals and Method(s) of evaluation
<p>Third key point:</p> <p><i>I can compare experimental and theoretical probabilities, including those based on equally likely outcomes and explain their findings using the correct language.</i></p>			Do all students understand the third key point?
<p>The advantage of us using a sample space is that we can answer other questions as well.</p> <p>For example</p>	Pupils answer questions on the research sheet.	<p>Teacher shows questions on the white board.</p> <p>What is the probability of getting the same colour on each spinner?</p> <p>What is the probability of not getting two the same?</p> <p>What is the probability of getting one red and one blue in no particular order?</p>	Did the students start to answer the questions immediately?
<p>A few weeks ago we asked you to think about this question.</p> <p>Show question. Lots of you gave different answers.</p> <p>Do you remember what you said last time?</p>	<p>Students read question.</p> <p>Fill in research sheet.</p>		Did the students remember what they said last time?
I want you to try to answer it today by drawing a sample space diagram for these spinners.	Students draw the sample space diagram.	Teacher helps them if necessary by writing on the row and column headings on the board.	Did the students draw the sample space correctly?
Now answer the question on your sheet.			
How many of you have changed your mind from last time?	Students put up their hands?		
What did you decide?			
<p>Ask supplementary question.</p> <p>What is the probability that the two spinners are a different colour?</p>	Students answer.		Did they all get a half?
<p>What would happen if the spinners look like this? (3 colours equal split) What is the probability of getting 2 yellows?</p>	Students look at the whiteboard and on their sheet.		Did the students all understand the question?
How are you going to decide?	Draw a sample space diagram.	Possibly set this for homework	Did they draw the sample space diagram with enthusiasm?
<p>What if the spinners look like the (3 colours unequal split 50:25:25 R:B:Y).</p>		This could be used at the start of the next lesson to see if the pupils can apply their new knowledge in a different context.	<p>Did the students match the sample space to the quarters?</p> <p>Did some match their sample space to the colours?</p>

Appendix 4: Proforma for Lesson Planning

Lesson Plan (see over for detailed lesson timings)				
School	Class	Teacher	Observers	Date
Topic to be covered:				
National Curriculum references:				
Pupils' prior knowledge/experience:				
Teaching and learning objectives:				
Resources required:				
Summary of lesson plan:				
Anticipated problems and difficulties:				
Specific classroom organisation/differentiation/IEP/SEN issues:				
Assessment arrangements:				

(60 minute lesson)

Lesson Timings Plan

Start time					
hrs	mins	Teacher/Class	Pupil/Class	Class/Working	Other Notes
5					
10					
15					
20					
25					
30					
35					
40					
45					
50					
55					
hrs	mins	KEY: NA: New Activity P: Praise Q: Question TR: Textbook Resource I: Interruption R: Reprimand A: Answer OR: Other Resource			
Finish time					

Board Work	
Review Points	
<ul style="list-style-type: none"> <input type="checkbox"/> Lesson well planned and prepared <input type="checkbox"/> Seating facilitates easy access of T/Ps, P/BB or OHP <input type="checkbox"/> Homework reviewed <ul style="list-style-type: none"> <input type="checkbox"/> Pupils showing/explaining solutions at board <input type="checkbox"/> Teacher or class pointing out mistakes <input type="checkbox"/> Mistakes used as teaching points and corrected <input type="checkbox"/> Reviewing other valid methods of solution <input type="checkbox"/> Interactive revision of previous lesson/topics <input type="checkbox"/> New concept introduced <ul style="list-style-type: none"> <input type="checkbox"/> Interactive discussion <input type="checkbox"/> Example worked on board with whole class <input type="checkbox"/> Immediate revision of forgotten/misunderstood topics <input type="checkbox"/> Individual work (exercises/activities) <ul style="list-style-type: none"> <input type="checkbox"/> Teacher continually taking note of what everyone is doing <input type="checkbox"/> Class kept together on exercises <input type="checkbox"/> Mistakes immediately pointed out to whole class <input type="checkbox"/> Extensions/challenges set for able pupils <input type="checkbox"/> Review/discussion of solutions by whole class 	<ul style="list-style-type: none"> <input type="checkbox"/> Relating to real-life contexts where appropriate <input type="checkbox"/> Relating to pupils' experiences where possible <input type="checkbox"/> Relating to other concepts where appropriate <input type="checkbox"/> Maths on B/B or white board correct, clear, precise, well set out <ul style="list-style-type: none"> <input type="checkbox"/> B/B instruments used effectively <input type="checkbox"/> Spoken maths clear, precise and correct <input type="checkbox"/> Mathematical logic stressed <input type="checkbox"/> Pupils allowed time to explain their thinking <input type="checkbox"/> Calculators and other IT used effectively <input type="checkbox"/> Summary of main points at end of lesson <input type="checkbox"/> Homework set <input type="checkbox"/> Homework clearly written on B/B or OHS <input type="checkbox"/> Enthusiasm of teacher <input type="checkbox"/> Pace of lesson <input type="checkbox"/> Variety of activities <input type="checkbox"/> Humour used <input type="checkbox"/> Class on task <input type="checkbox"/> Class progression
Rough Plan of Classroom	Points to make
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Board (Focus)</div>	<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: center;">+</div> <div style="border-left: 1px dashed black; width: 1px; height: 100%;"></div> <div style="text-align: center;">-</div> </div>

Appendix 6: Proforma for Lesson Review

Lesson Review				
School	Teacher	Class	Date	Time
Year	Level	Topic		
Observers of lesson (<i>initials</i>)		Observers of video (<i>initials</i>)		Participants in review (<i>initials</i>)
Summary of Key Points Noted				
+				
-				
Agreed actions				

Appendix 7: Protocols for Within-School Lesson Study

Getting Started
<ul style="list-style-type: none"> • Get the agreement of all teachers of mathematics to participate in the initiative • Select a leader to drive the initiative and deal with issues that arise • Agree an overarching aim of acknowledged importance for the research focus of lesson study • Turn this overarching aim into specific, measurable objectives that can eventually be used to evaluate the success of your research lessons over the next three years • The group size should be four or five (but three would be possible) • Choose the participants in each group (giving careful thought to the mix of the group, their experience, etc.) • Design a schedule of sessions for each of the groups so that one cycle is covered in no more than one term (this of course has to be negotiated with the senior management so that any required cover, etc. is organised)
Planning the Research Lesson
<ul style="list-style-type: none"> • Agree a topic of acknowledged importance to the group and document the reasons behind the choice • Map out the unit of work into a series of lessons and then choose which lesson is the key to the unit • Identify the learning goals for both the unit and the research lesson • Encourage everyone to think about the most effective way of teaching the lesson • Two teachers take responsibility for producing a draft lesson plan based on the joint decisions of the group • One teacher takes responsibility for producing the final lesson plan
Teaching and Observation
<ul style="list-style-type: none"> • Include the students in the lesson study process so that they understand its purpose, their roles and the roles of the observers • Follow the agreed lesson plan • All participants in the group should observe the lesson, taking notes of key points or incidents • Observers should not obstruct the flow of the lesson • Observers should not review or evaluate the lesson until the scheduled session
Lesson Review
<ul style="list-style-type: none"> • One of the observers should act as chair • The teacher should reflect on their lesson first • Each observer should have the chance to give their reflections (in a positive way) and to questions the teacher on action taken
Future Policy
<ul style="list-style-type: none"> • Any recommendations or issues for school/departmental policy resulting from a cycle of the group's work should be discussed at a full staff/departmental meeting • The overarching aim should not be changed for at least three years • Lesson study groups remain fixed for the whole school year • Lesson study groups periodically meet together to share their findings • The leader should consider making changes to the membership of each group after each year of activity

Appendix 8: Understanding Japanese Mathematics Lessons

As we mentioned earlier we firmly believe that lesson study can transform mathematics teaching in your school provided that, at the same time, you consider exactly what you are trying to achieve through your lessons. The success of Japanese mathematics, however, depends not only on lesson study as a means of sharing professional expertise but also on a proven method of teaching mathematics.

In this section we will explain more about this method.

As Stigler and Hiebert (**reference (4)**) note, many Japanese mathematics lessons follow a similar sequence: reviewing the previous lesson; presenting the problem of the day; students working individually or in groups; discussing solution methods; and, highlighting and summarising the main point.

After reviewing the previous lesson as appropriate, the teacher's role changes as the lesson progresses through four distinct phases.

Phase One: Presentation of the problem (about 10 minutes)

- The teacher presents the problem in a way that the students can easily understand and know what is expected of them.
- The students determine what the problem is about by reading it themselves, listening to the teacher's instructions and discussing it amongst themselves. They check similarities and differences between what they already know and what they need to learn today and begin to develop a perspective on ways of tackling the problem.

Phase Two: Developing a solution (about 15 minutes)

- The students think about the problem on their own and try to find solutions on their own. They may be asked to then share ideas with their partner.
- The teacher purposefully walks around looking at the students' work, making notes

and deciding in which order he will ask the students to present their ideas in phase three. The teacher in general does not guide the students apart from giving hints to students who cannot make progress on their own. In this way the maths being created belongs to the students.

Phase Three: Progress through discussion (about 10 minutes)

- The teacher asks three to five students who used different methods (or got different answers depending on the type of openness being used) to explain their approaches to the rest of the class. The teacher remains neutral to the ideas.
- The students listen to the explanations and try to reach a common understanding of better solutions by discussing the strong and weak points of each approach proposed and identifying what they have in common.

Phase Four: Summarising (about 10 minutes)

- The teacher summarises the group findings and in particular emphasises the important points addressed in the lesson. The teacher will challenge the students with similar or developmental problems for homework.
- The students often write down what they have learned in their journals.

One of the main differences between Japan and the West is that the teachers not only share a common understanding of this approach but that they also use specific pedagogical terms to describe and discuss their roles in the process.

Fernandez and Yoshida (**see reference (17)**) explain that:

'Through working together, particularly during lesson study, Japanese teachers have developed a large array of technical terms and expressions to help them communicate more effectively. This

professional language is widespread and specific to teaching in that none of the terms or expressions can be found in a regular Japanese dictionary.'

For example, following their work:

Hatsumon refers to the teacher's questions or actions designed to help provoke students' deep thinking at a particular point of the lesson. This often occurs in phase one of the model.

Kikanjunshi, literally means to walk around between desks and inspect students' work but it also involves collecting notes on the students ideas and thinking about who to ask to come up to present their work and in what order. This takes place in phase two of the model.

Neriage describes the dynamic and collaborative nature of a whole class discussion in the lesson. The term describes the process of 'polishing' students' ideas and getting an integrated mathematical idea through whole-class discussion. Japanese teachers regard 'neriage' as critical for the success or failure of the entire lesson. As a result of this approach Japanese students see themselves explicitly as learners and are aware of their developing skills as it is they, rather than the teacher, who are perceived to be 'doing the maths'.

Matome refers to the teacher or one of the students summarising what the class have learned today. Japanese teachers think this is indispensable to a successful lesson.

The Open Approach

Problem solving lessons are often used as the research lesson in Japan and considerable care is taken in choosing an appropriate problem. It is important to remember that a suitable problem will be one for which the students have no known method of solving. In other words the students are solving the problem not to apply mathematics but to learn new mathematics.

They first determine if the problem is appropriate by asking three questions:

- Is the problem rich in mathematical content and valuable mathematically?
- Is the mathematical level of the problem appropriate for the students?

- Does the problem include some mathematical features that lead to further mathematical development?

Much of the current Japanese approach to problem solving can be traced back to the research carried out between 1971 and 1976 by Japanese researchers in a series of projects on methods of evaluating higher-order-thinking in mathematics education. As this work progressed the researchers led by Shigeru Shimada 'became aware that lessons based on solving open-ended problems as a central theme have rich potential for improving teaching and learning' (see reference (21)). 'Open ended' or 'incomplete' problems were defined as problems that have multiple correct answers.

Later this work became known as simply the 'Open Approach' and included three types of openness solving namely:

1. Process is open – where the learning comes from studying the different ways of solving a problem.
2. End product is open – where the learning comes from studying the different answers.
3. Ways to develop are open – where the learning comes from the students using the initial problem to generate new problems of their own.

The open-ended approach begins by choosing a suitable problem that all students will be able to access. Shimada (reference (21)) explains that as the approach places 'special emphasis on the mathematical thinking of individual students', it is important that the teacher remains neutral. He warns that 'the openness is lost if the teacher proceeds as though only one method is presupposed as the correct one.'

Sawada (reference (22)) explains that:

'The teacher then makes use of the diversity of approaches to the problem in order to give students experiences in finding or discovering new things by combining all the knowledge, skills and mathematical ways of thinking they have previously learned.'

He goes on to say that this model of teaching combines individual work and whole class

discussion. 'It is crucial to this approach to proceed from individual learning to group learning.'

Sawada (see reference (22)) points out that the advantages of using this model are:

1. Students participate more actively in the lesson and express their ideas more frequently.
2. Students have more opportunities to make comprehensive use of their knowledge and skills.
3. Even low achieving students can respond to the problem in some significant ways of their own.
4. Students are intrinsically motivated to give proofs.
5. Students have rich experiences in the pleasure of discovery and receive the approval of fellow students.

Here are some examples of open ended questions.

Geometry

We want to enlarge this rectangle to double its dimensions. What drawing methods can you discover to do this? Draw your figure by as many different methods as you can.



Explain your method in words.

Data Analysis

In a school gymnastics competition, Jenna and Kim were the top two competitors. The five judges gave them the following scores.

	Jenna	Kim
Judge 1	8	9
Judge 1	8	9
Judge 2	6	9
Judge 3	10	7
Judge 4	9	8
Judge 5	7	7

Give a reason why Jenna might have been declared the winner.

Can you convince me that Kim should have been the winner?

Number

Perfect numbers are positive integers in which the divisors of the number (excluding the number itself) add up to the number. The first perfect number is 1 and the second is 6 (as $6 = 3 \times 2$ and $6 = 1 \times 6$ and $3 + 2 + 1 = 6$).

David has suggested that a formula to generate perfect numbers is

$$2^{n-1}(2^n - 1) \text{ for } n = 1, 2, \dots$$

Do you agree with him?



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